Far-to-Near extrapolation ~ Flux covariance method and F/N ratio method ~

Mark Hartz, A.Murakami

- Introduction for flux covariance method and F/N ratio method
- Method of calculation including rebinning
- Covariance from error sources for flux covariance, F/N ratio

Flux covariance method

- For 2011 analysis, the plan is to include the ND280 nu_mu measurement as part of a ”global” likelihood

S. Oser

Global Oscillation Likelihood

$$\ln L(\vec{o},\vec{b},\vec{x},\vec{n},\vec{s}) = \ln L_{beam}(\vec{b}) + \ln L_{xsec}(\vec{x}) + \ln L_{ND280}(\vec{b},\vec{x},\vec{n}) + \ln L_{SK}(\vec{b},\vec{x},\vec{s},\vec{o})$$

Function of oscillation (*o*), beam/flux (*b*), xsec (*x*), ND280 sys (*n*), and SK sys (*s*) parameters. Ultimately we only care about the oscillation parameters *o*, and we'll marginalize over the other nuisance parameters (*b*,*x*,*n*,*s*).

The flux parameters *(b)* in the global likelihood scale the flux in a given neutrino flavor, energy bin

$$\phi_i' = b_i \phi_i$$

Prior Constraint on the Flux Parameters

- In the global likelihood, there is a prior constraint on the flux parameters:

$$-2\ln[L_{beam}(\vec{b})]=\sum_{i}\sum_{j}(1-b_i)V^{-1}_{i,j}(1-b_j)$$

- *V* is the prior covariance for the flux bins based on the hadron production, proton beam and alignment errors
- Since the *b* are factors that scale the flux in a given bin, *V* is the fractional covariance

Far/Near ratio method

K2K method

Normalization term

$$N_{\exp} = N_{\exp}(f_{\Phi}, f_{nQE}, f_{F/N}, f_{n11}, \Delta m^2, \sin^2 2\theta)$$

$$= \frac{N_{SK}^{MC}(f, P_{osc})}{N_{KT}^{MC}(f)} N_{KT}^{DATA}$$

HARP

tuned by Cho param.

ND measurement

$$= \frac{\sum_{i,j} f_{\Phi i} f_{F/Ni} \cdot \Phi_{ND}^{MC}(E_i) P(\Delta m^2, \sin^2 2\theta) \cdot (f_j \sigma_{ij}) \cdot \varepsilon_{ij}^{SK}}{\sum_{i,j} f_{\Phi i} \Phi_{KT}^{MC}(E_i) \cdot (f_j \sigma_{ij}) \cdot \varepsilon_{ij}^{KT}}$$

ND measurement

i :energy, j :mode

shape term

$$\Phi_{\exp}^{SK}(E_i) = f_{\Phi i} f_{F/Ni} \cdot \Phi_{ND}^{MC}(E_i) P(\Delta m^2, \sin^2 2\theta) \cdot (f_j \sigma_{ij}) \cdot \varepsilon_{ij}^{SK}$$

- ND fit results ($f_{\Phi i}$, f_j and error matrix) are propagated into SK fit.
  - K2K fits (p_μ, θ_μ) distribution.
- Common flux uncertainties between ND and SK (hadron production etc..) are expected to be cancelled (in $f^{F/N}$ term)

Calculating the Covariance Matrix

- For independent errors sources, a total covariance can be derived by adding the covariance from each error source:

$$V_T = V_{pion} + V_{kaon} + V_{pbeam} + \ldots$$

- Two methods for calculating the covariance for a give error source

One sigma method:

- Fractional changes in the flux are calculated for +/- 1 sigma deviations of a single underlying parameter, e.g. off-axis angle
- Covariance is calculated from the fractional changes to the flux

$$V_{i,j} = \frac{\Delta\phi_i^{+1\sigma}\Delta\phi_j^{+1\sigma} + \Delta\phi_i^{-1\sigma}\Delta\phi_j^{-1\sigma}}{2}$$

Fractional change in flux (or F/N)

Throwing Method

- Many underlying parameters with their own covariance are varied at one time, e.g. NA61 pion multiplicity bins
- Underlying parameters are thrown many times according to their own covariance
- For each throw the flux is reweighted with the thrown parameter set
- The covariance is calculated from the reweighted flux in N throws

$$V_{i,j} = \frac{1}{N-1} \frac{\sum_{k=1}^{N} (\phi_i^{nom} - \phi_i^k)(\phi_j^{nom} - \phi_j^k)}{\phi_i^{nom} \phi_j^{nom}}$$

Flux covariance Binning

• The covariance matrices are originally calculated with the energy binning:

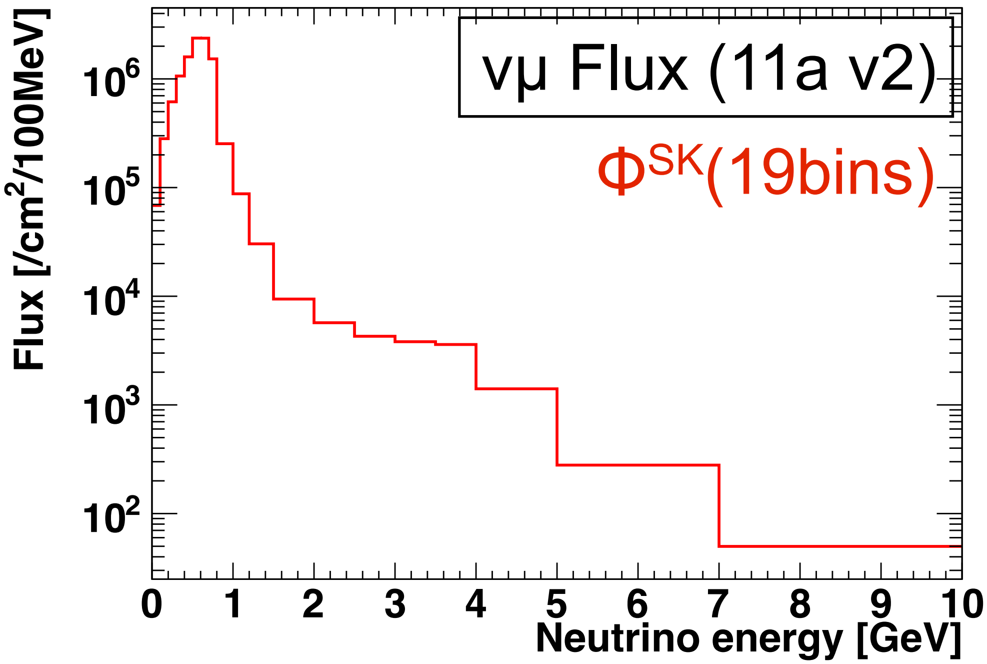
```
int nbins = 19;
double bins[20] = {0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0,
                   1.2, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 5.0, 7.0, 10.0}
```

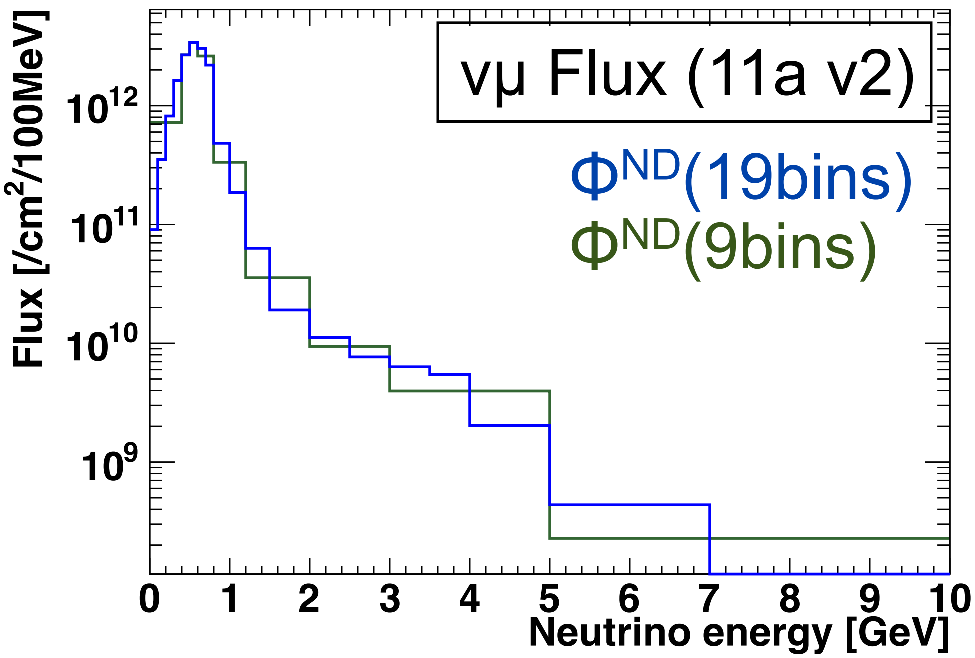
• For some applications, we may want a covariance with a coarser binning. For example, the binning used in the ND280 nu_mu fit is:

```
//ND280 binning in E_nu (GeV)
 int nbins_nd = 9;
 double bins_nd[10] = {0.0, 0.4, 0.5, 0.6, 0.8, 1.2, 2.0, 3.0, 5.0,10.0};
```

F/N Binning

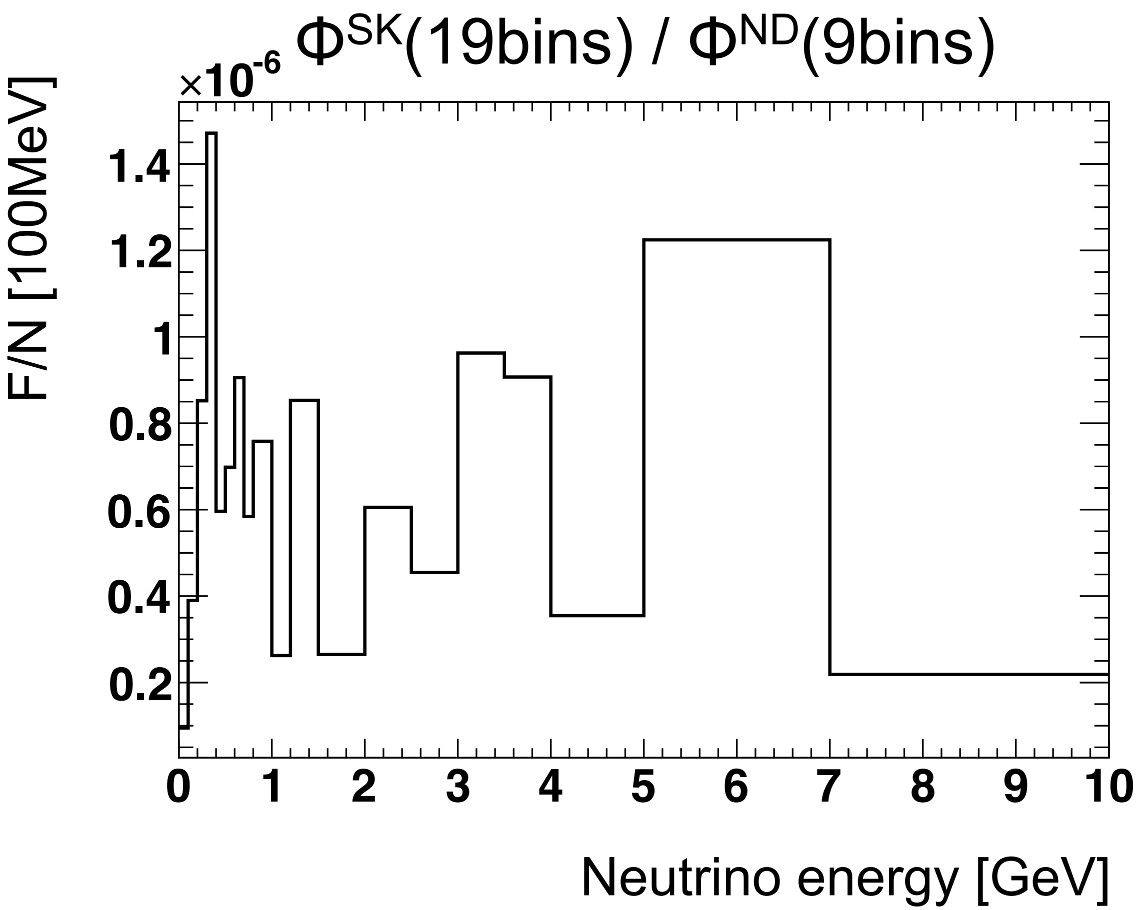
- Calculate F/N with the following binning to extrapolate the current ND280 fitting results to SK
  - SK(19bins) / ND(9bins)
    - Definitions of "19bins", "9bins" are same as previous page.
- Calculate the flux ratio : the SK flux in each bin is divided by the ND5 flux in the bin which covers the SK bin.
  - $F/N(0\sim0.1GeV) = \Phi^{SK}(0\sim0.1GeV) / \Phi^{ND}(0\sim0.4GeV)$
  - $F/N(0.1\sim0.2GeV) = \Phi^{SK}(0.1\sim0.2GeV) / \Phi^{ND}(0\sim0.4GeV)$

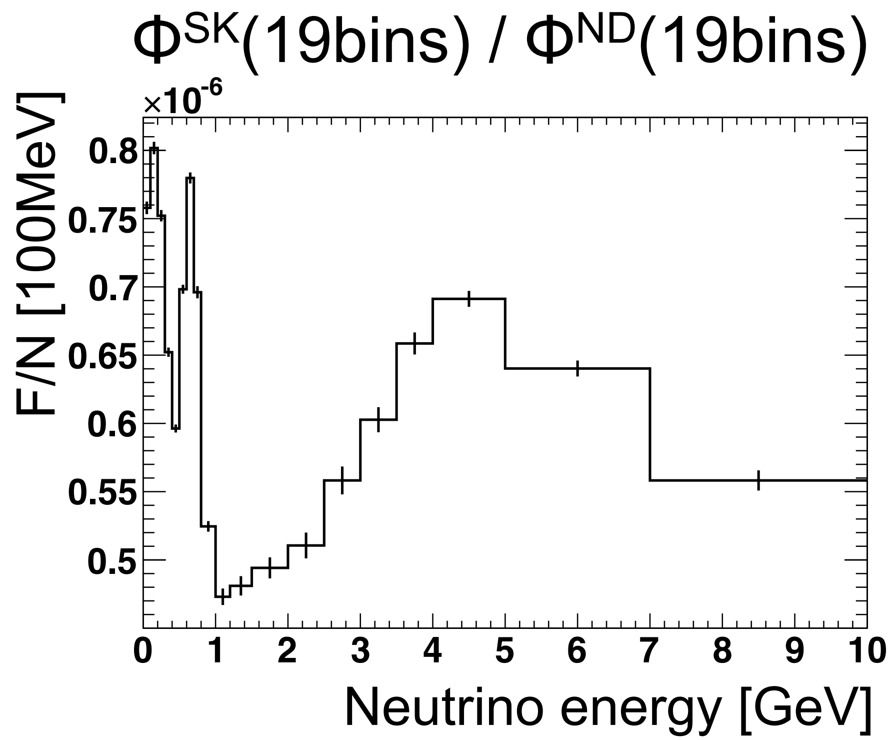




F/N ratio

Calculate F/N ratio with νμ flux shown according to the previous page.





Error Sources for flux covariance

- ☑ **Pion multiplicity** – update with Mark-san' report
- ☑ **Kaon multiplicity** – updated with Slavic's report
- ☑ **Secondary nucleon** – from 1 sigma errors used in Run 1+2 analysis.
- ☑ **Off-axis angle** – from 1 sigma errors calculated for 0.44 mrad uncertainty
- ☑ **Proton beam** – estimated by throws using the reweighting based on proton beam uncertainty (Run1&2).
- ☑ **Production cross sections** – from throws of quasi-elastic uncertainty in different regions, same as 2010a nu_mu shape error method
- ☐ **Horn & target alignment** – from 1 sigma errors estimated for 2010a analysis, needs to be added
- ☐ **Absolute horn current** – from 1 sigma errors estimated for a 5 kA change, needs to be added
- ☐ **Horn field asymmetry** – based on results of field measurements, needs to be added

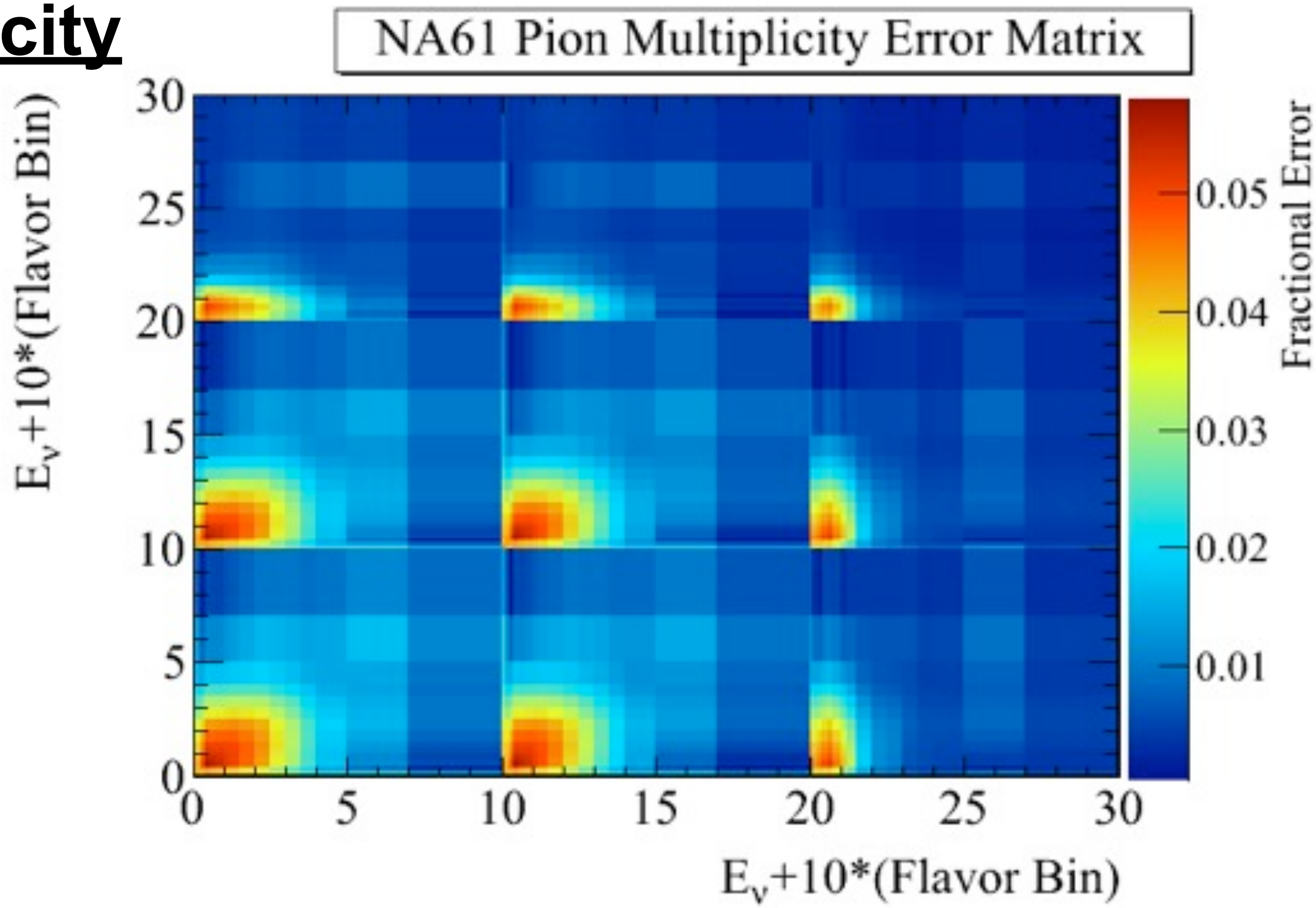
Error Sources for F/N

- ☑ **Pion multiplicity** – same as Run 1+2 (not updated with Mark-san's)
- ☑ **Kaon multiplicity** – updated with Slavic's report
- ☑ **Secondary nucleon** – from 1 sigma errors used in Run 1+2 analysis.
- ☑ **Off-axis angle** – from 1 sigma errors calculated for 0.44 mrad uncertainty
- ☑ **Proton beam** – estimated by throws using the reweighting based on proton beam uncertainty (only Run 1).
- ☑ **Production cross sections** – from throws of quasi-elastic uncertainty in different regions, same as 2010a nu_mu shape error method
- ☐ **Horn & target alignment** – from 1 sigma errors estimated for 2010a analysis, needs to be added
- ☐ **Absolute horn current** – from 1 sigma errors estimated for a 5 kA change, needs to be added
- ☐ **Horn field asymmetry** – based on results of field measurements, needs to be added

Pion Multiplicity (flux covariance)

NA61 Pion Multiplicity

- Covariance is updated with studies shown by Mark-san.
  - Quadratic form for NA61 PID correlations
  - BMPT extrapolation for error beyond NA61 phase space



Error matrix:

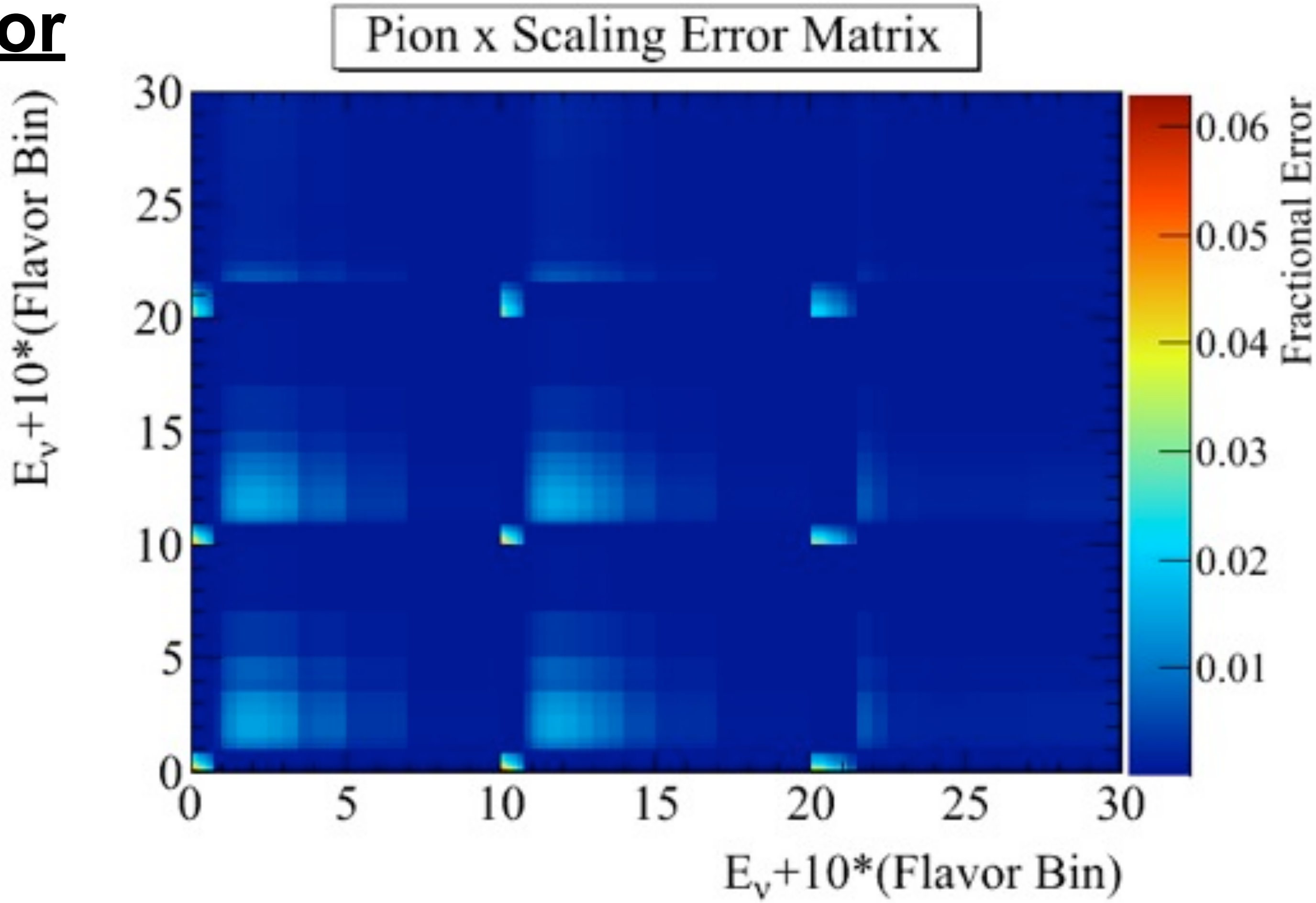
$$E_{i,j} = sign(V_{i,j})\sqrt{|V_{i,j}|}$$

Flavor bins are 0 = ND5 nu_mu
1 = SK nu_mu
2 = SK nu_e

Pion Multiplicity (flux covariance)

Pion x Scaling Error

- From the difference between two tertiary pion tuning methods:
  - xF-pT scaling with NA61 only
  - xR-pT scaling with NA61 and E910

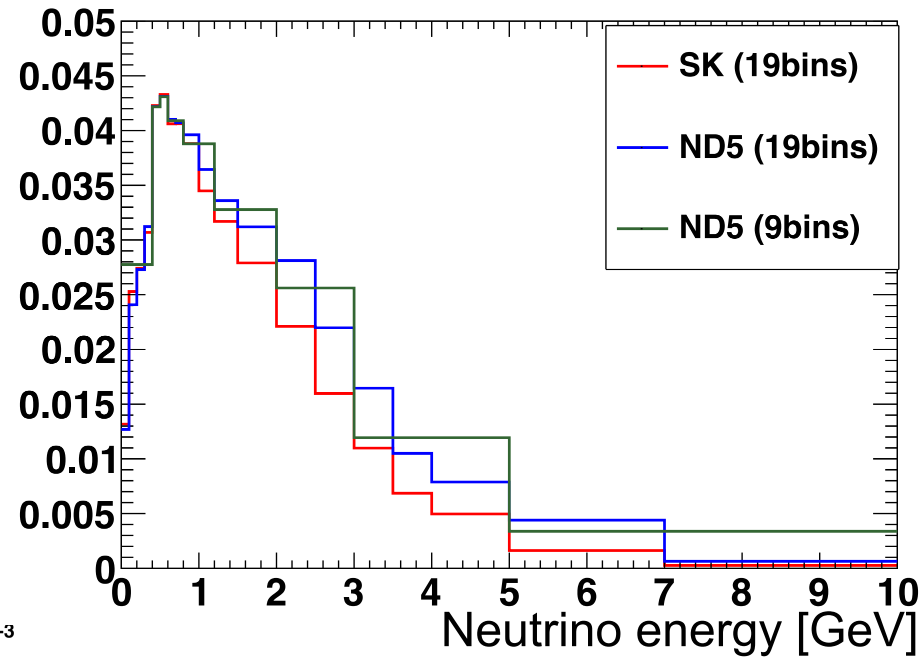


Flavor bins are 0 = ND5 nu_mu
1 = SK nu_mu
2 = SK nu_e

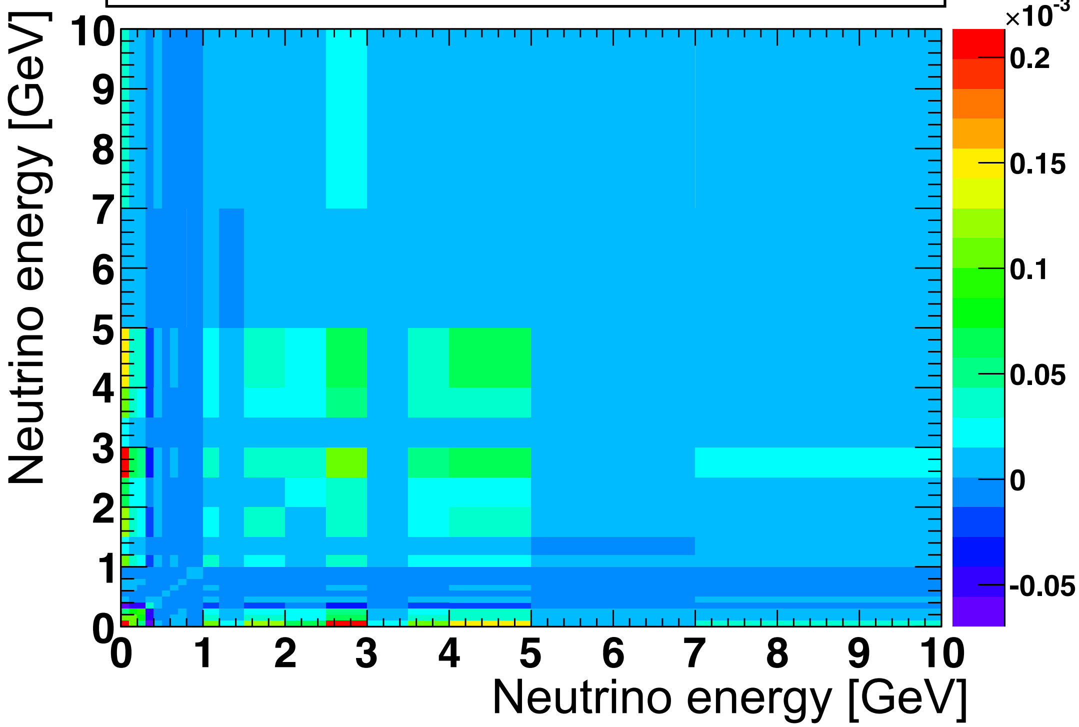
Pion Multiplicity (F/N) SK:19bins / ND:9bins

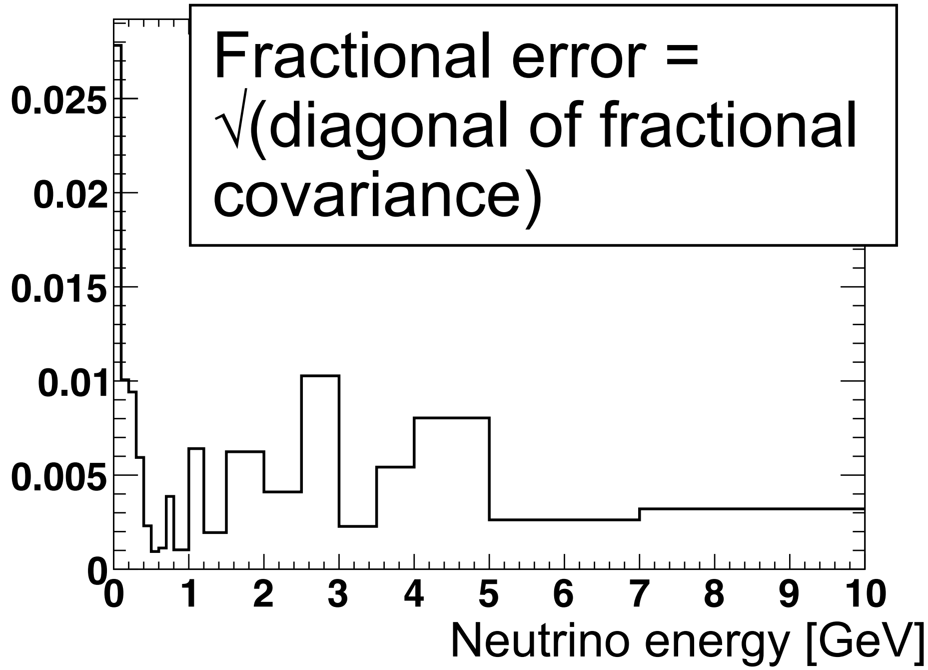
Not include Mark's update

Fractional error for SK, ND5 →



Fractional covariance matrix



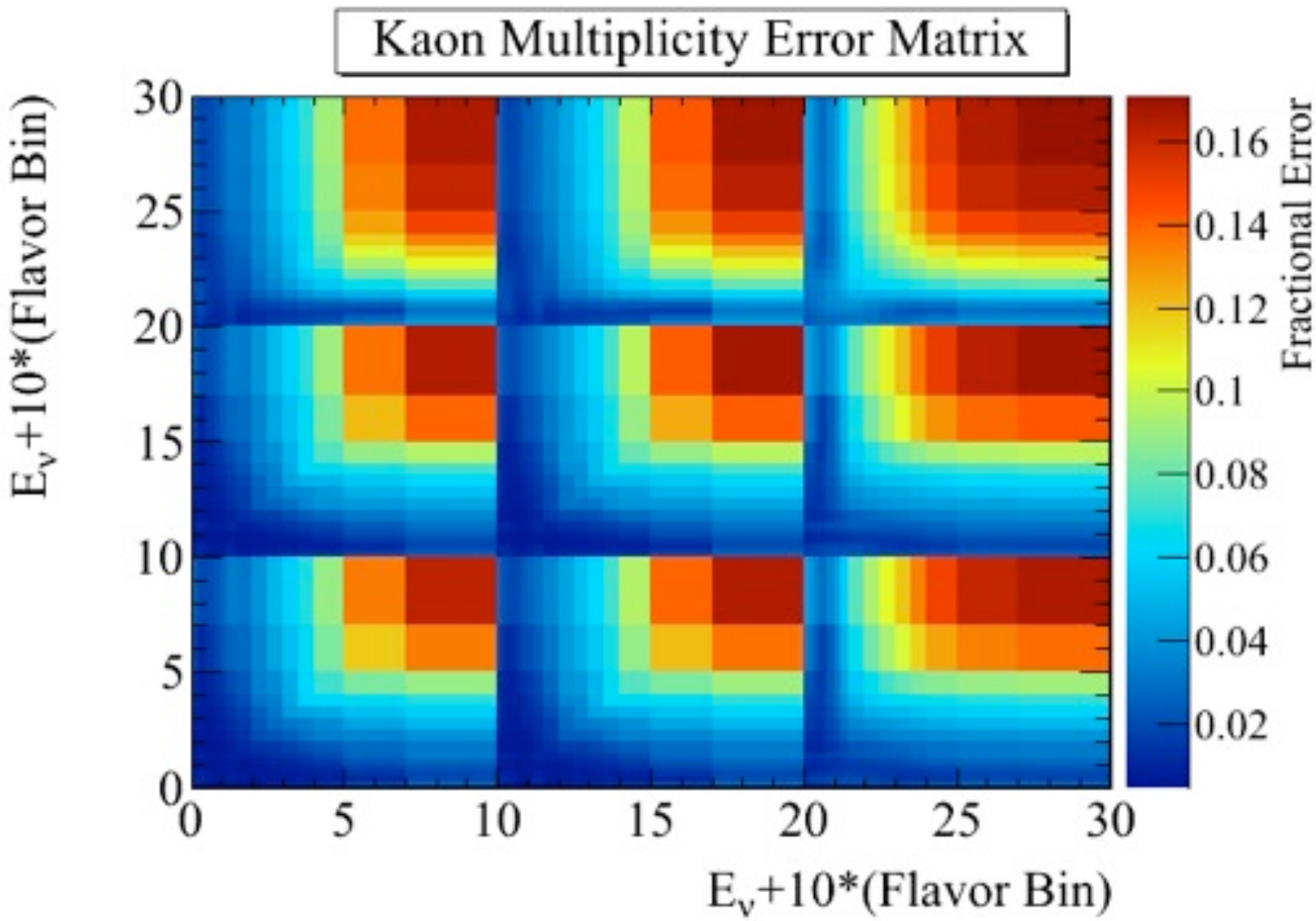


Fractional error = √(diagonal of fractional covariance)

Kaon Multiplicity (flux covariance)

Using error matrix shown by Slavic' study (which reported today)

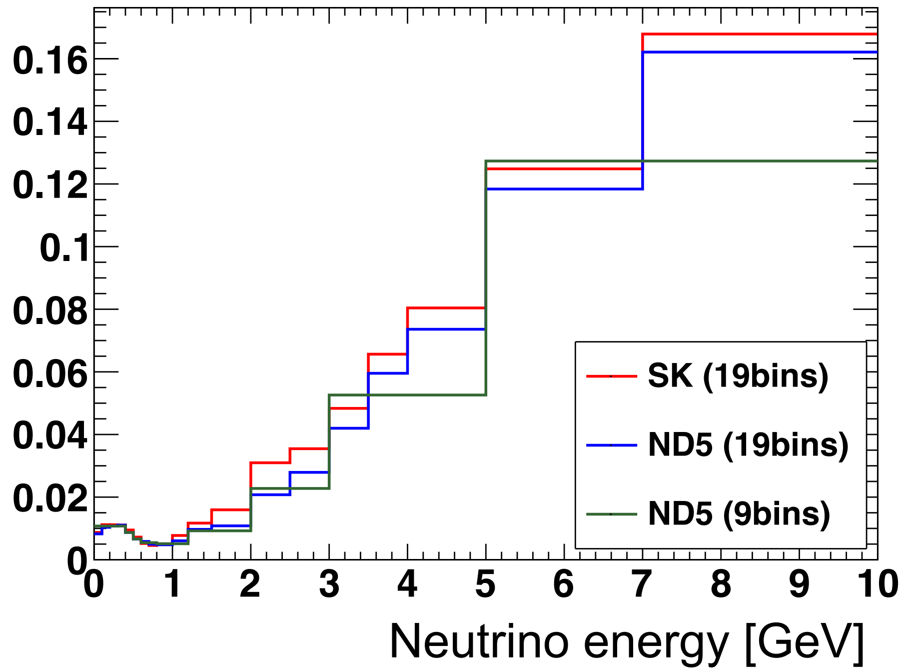
Error matrix: $E_{i,j} = sign(V_{i,j})\sqrt{|V_{i,j}|}$



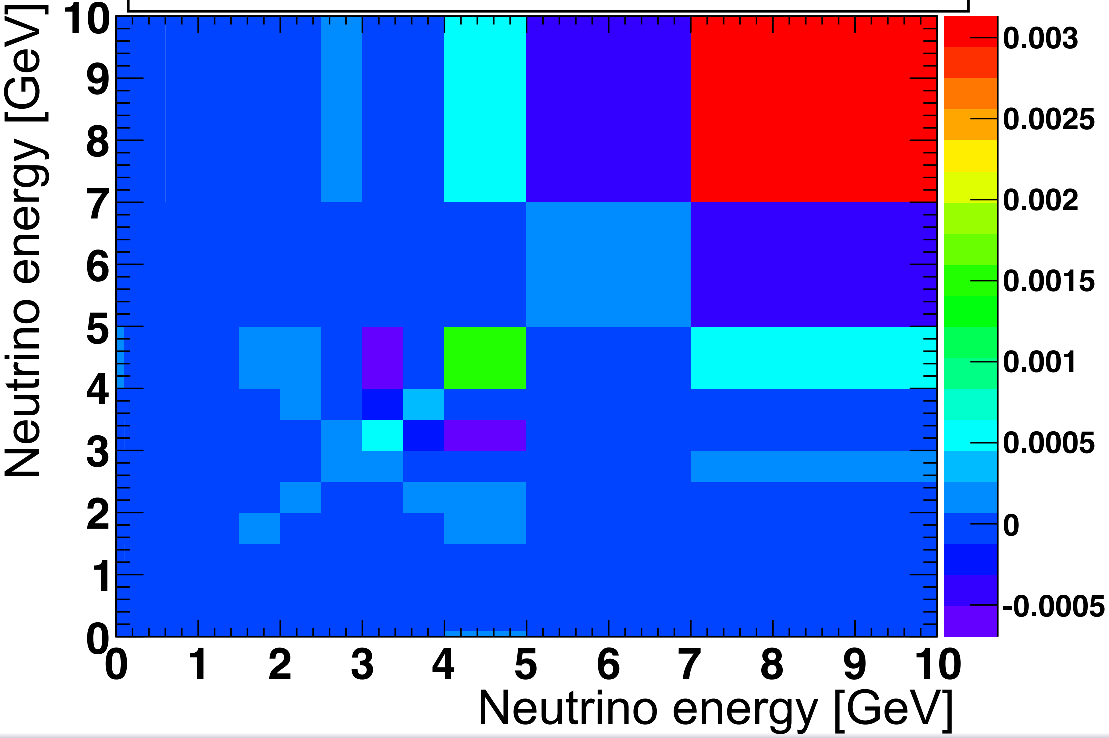
Flavor bins are 0 = ND5 nu_mu
1 = SK nu_mu
2 = SK nu_e

Kaon Multiplicity (F/N) SK:19bins / ND:9bins

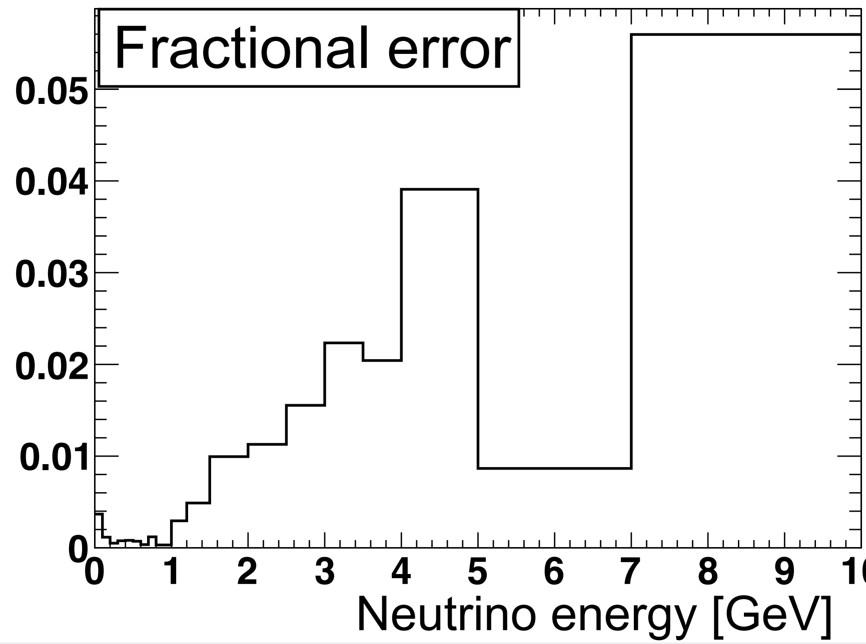
Fractional error for SK, ND5 →



Fractional covariance matrix

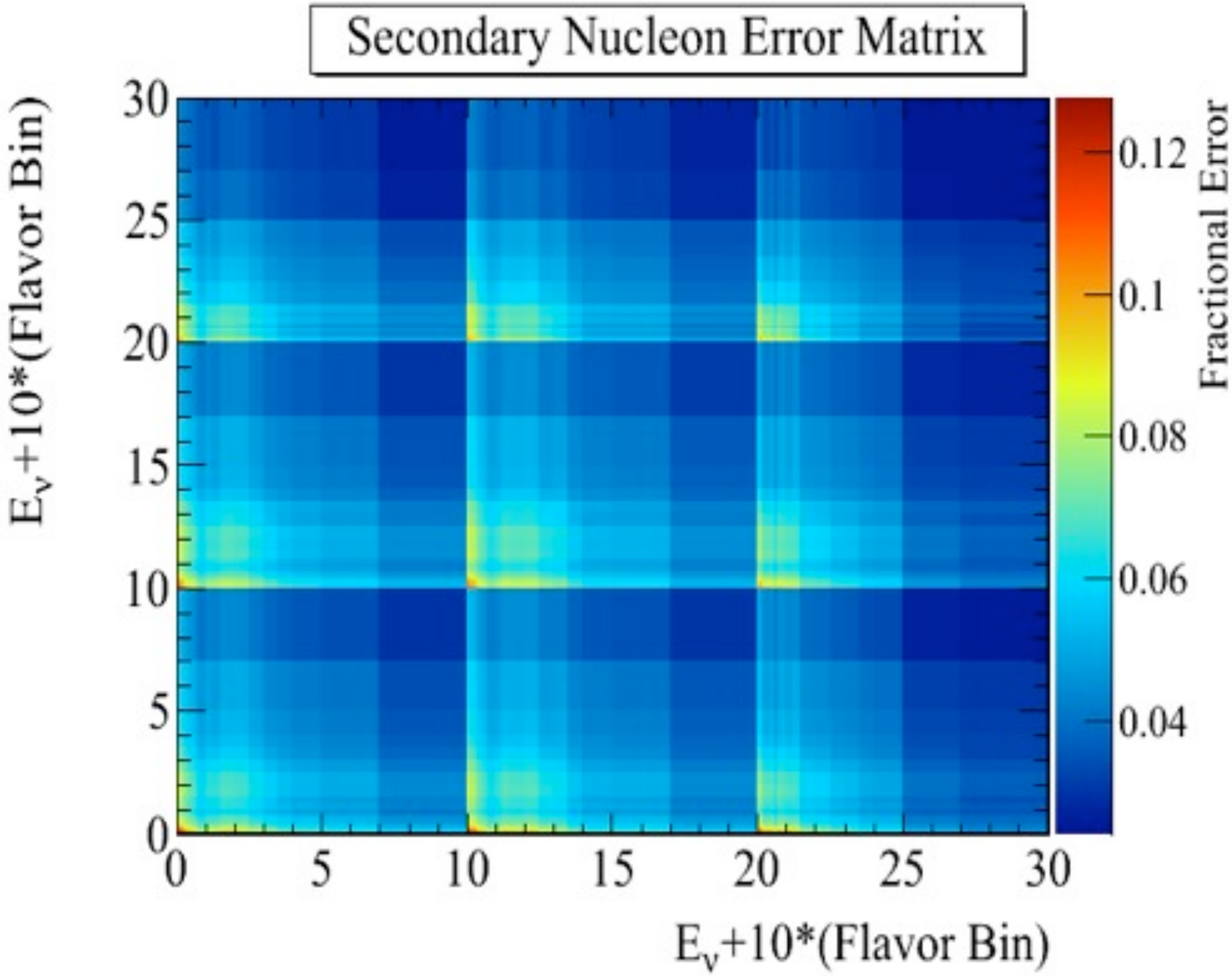


Fractional error



Secondary Nucleon (flux covariance)

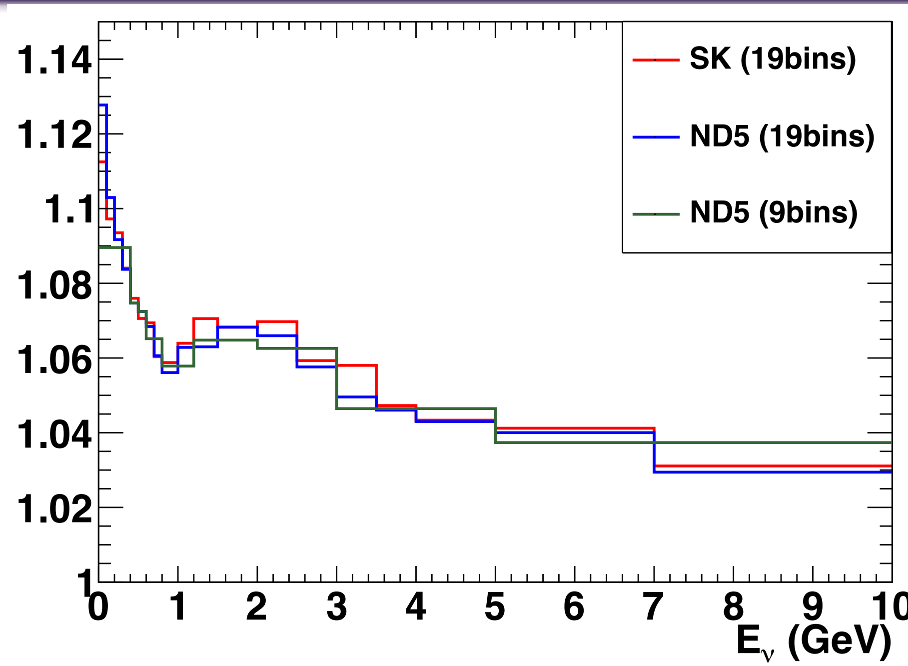
- Covariance from 1 sigma deviations used in the Run 1+2
  - For $x_F<0.9$ – difference between FLUKA and reweighting with Eichten and Allaby
  - For $x_F>0.9$ – factor of 2 change in rate with leading baryon conservation constraint

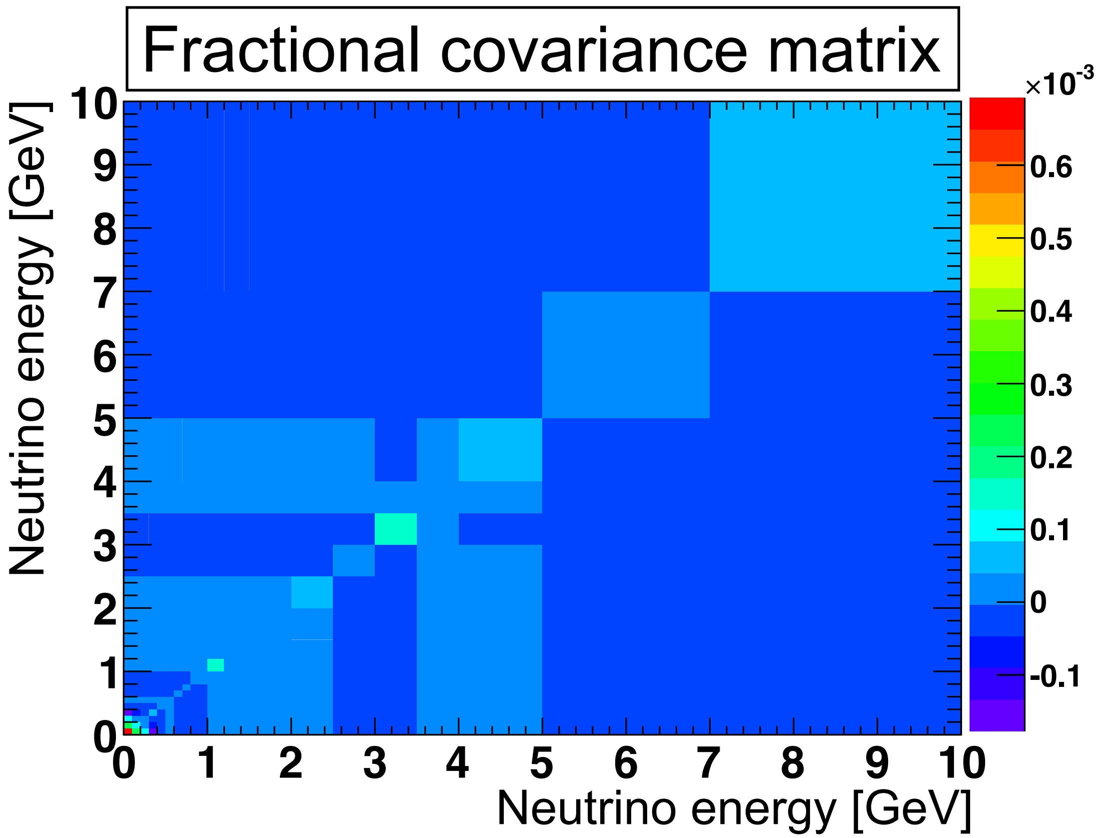


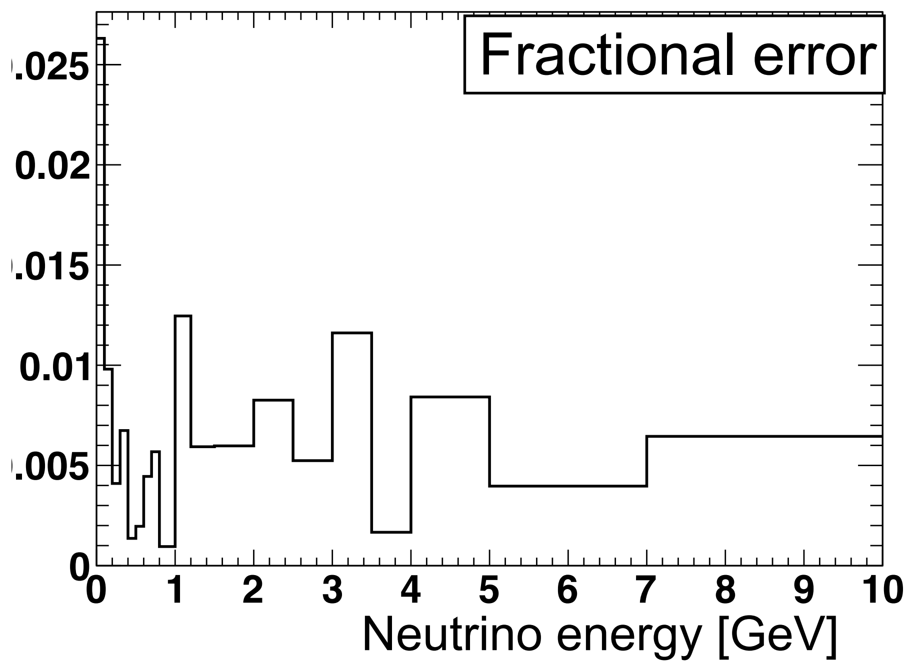
Flavor bins are 0 = ND5 nu_mu
1 = SK nu_mu
2 = SK nu_e

Secondary Nucleon (F/N) SK:19bins / ND:9bins

Ratio of 1 sigma changed to nominal →

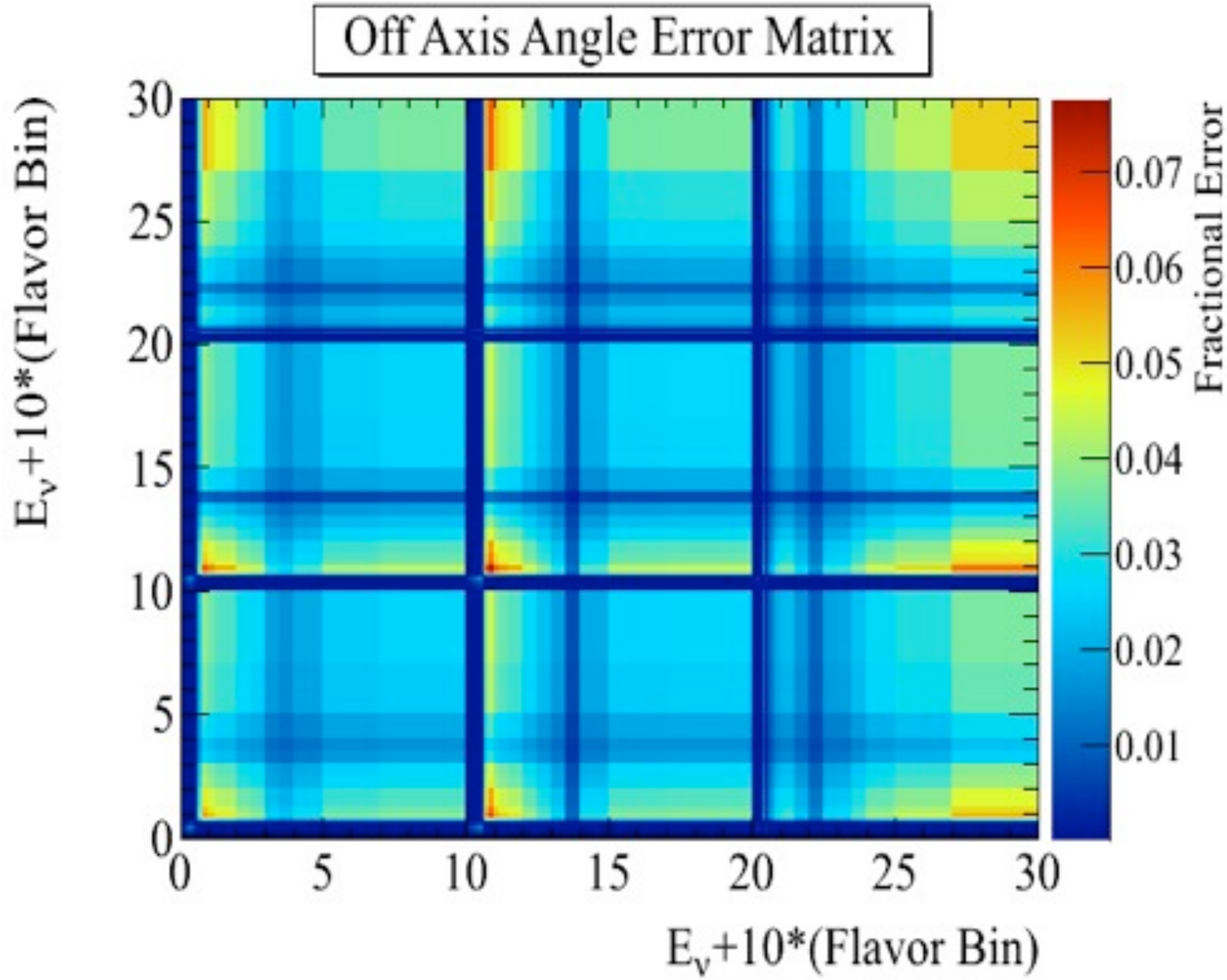






Off Axis Angle (flux covariance)

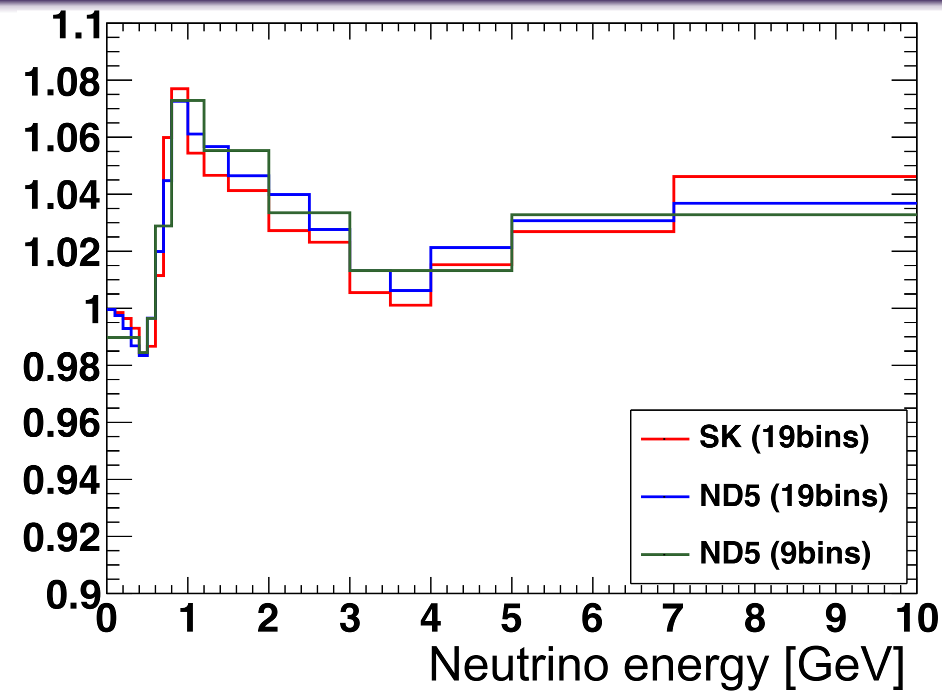
- From 1 sigma flux deviations used in Run 1+2 analysis
  - 0.44 mrad uncertainty in the neutrino beam direction

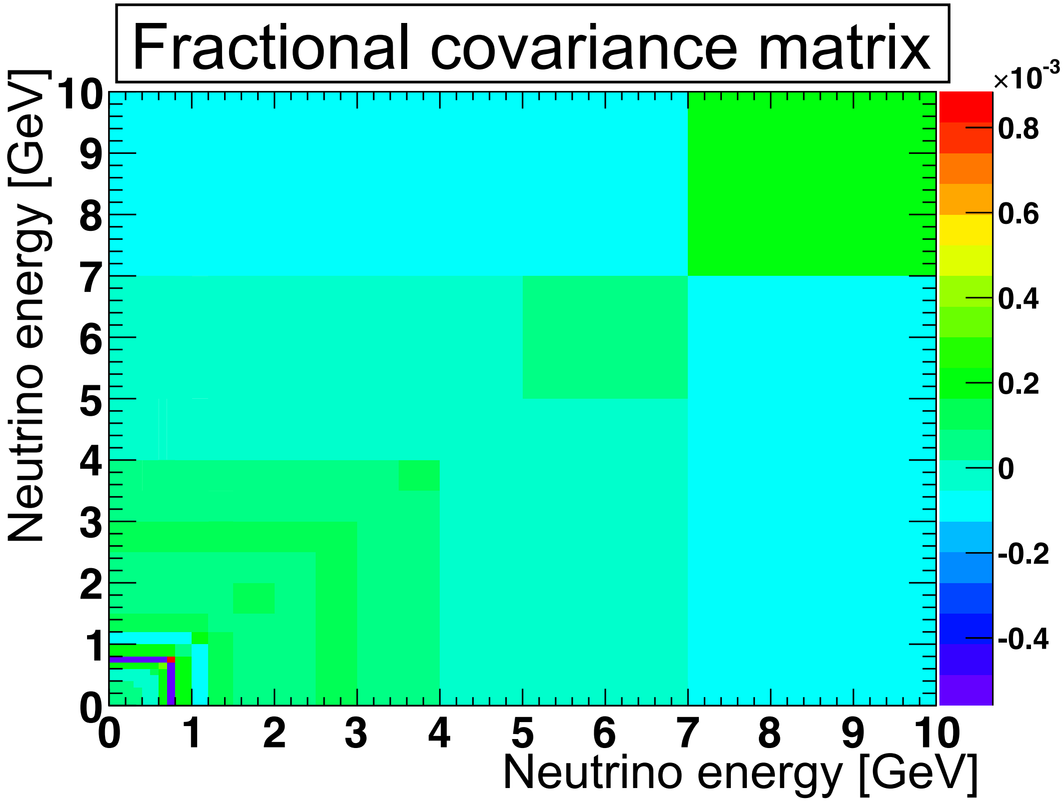


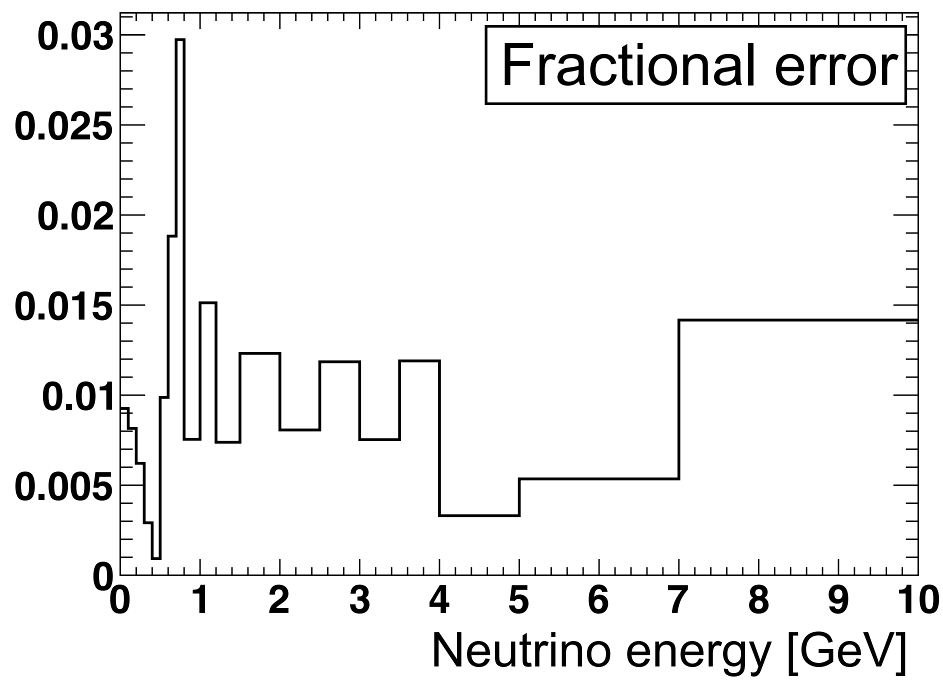
Flavor bins are 0 = ND5 nu_mu
1 = SK nu_mu
2 = SK nu_e

Off Axis Angle (F/N) SK:19bins / ND:9bins

Ratio of +1sigma changed flux to nominal one →







Proton Beam

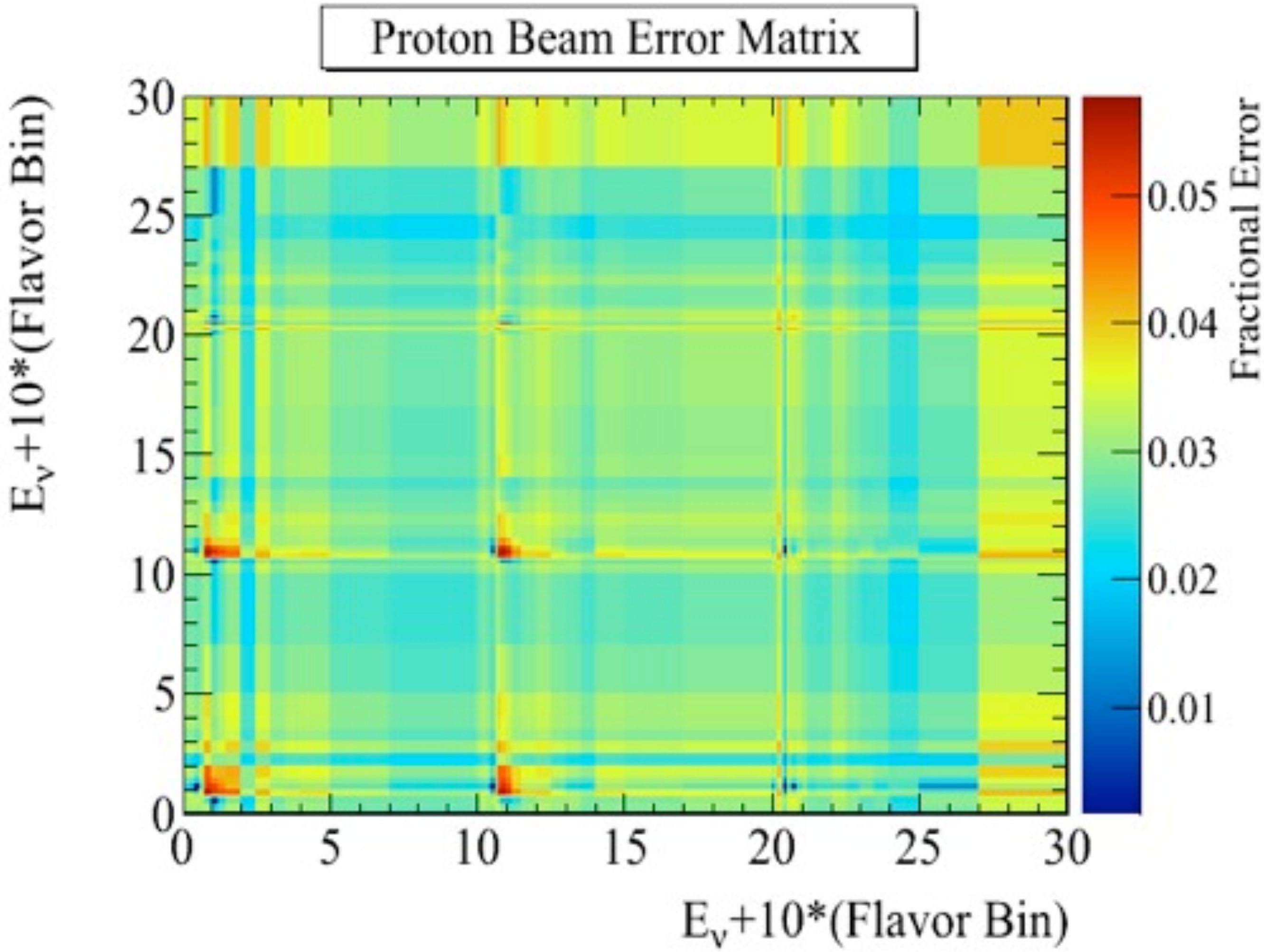
- Currently using error calculated with the proton beam reweighting implemented in T2KReWeight
  - See slides from beam workshop: http://www.t2k.org/beam/meetings/beamanawc/wctalks/t2kreweight/at_download/file
- Flux covariance : larger of Run 1 or Run 2 errors
- F/N ratio : only Run1 (only for y-y' phase space, same as 2010a)

Uncertainty of the primary beam optics(TN054)

	Run I	Run II
width in X (mm)	0.11	0.26
width in Y (mm)	0.97	0.82
Twiss α in X	0.32	0.26
Twiss α in Y	1.68	0.49
position in X(mm) (x)	0.38	0.27
position in Y(mm) (y)	0.58	0.62
angle in X (mrad) (x')	0.056	0.064
angle in Y (mrad) (y')	0.286	0.320
$\mathrm{cov}(x, x')$	0.011	0.013
$\mathrm{cov}(y, y')$	0.065	0.079

Proton Beam (flux covariance)

Place holder error matrix from larger of Run 1 or Run 2 errors

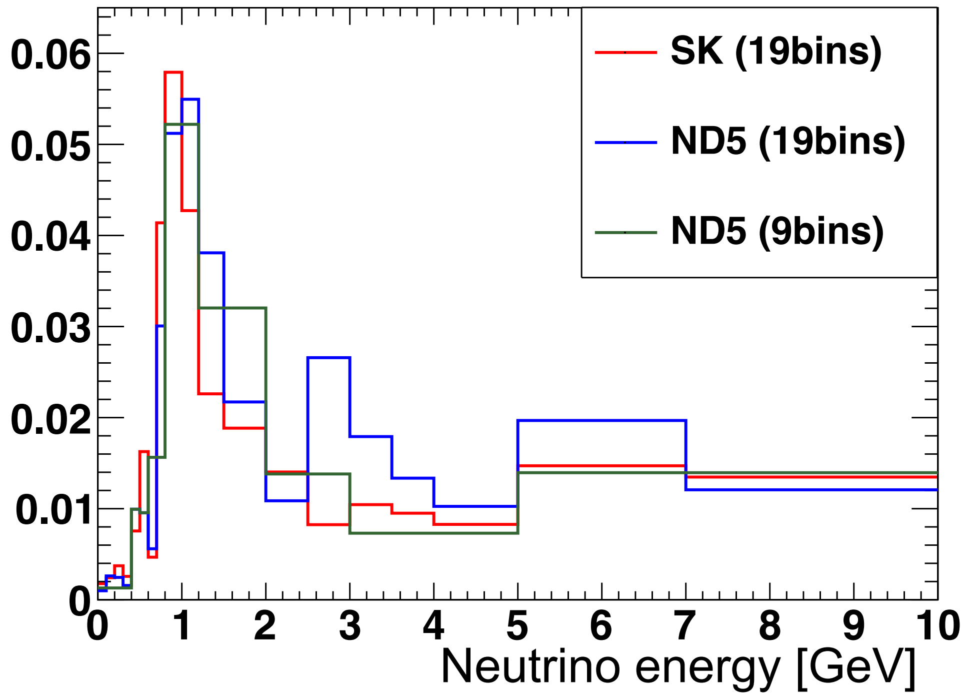


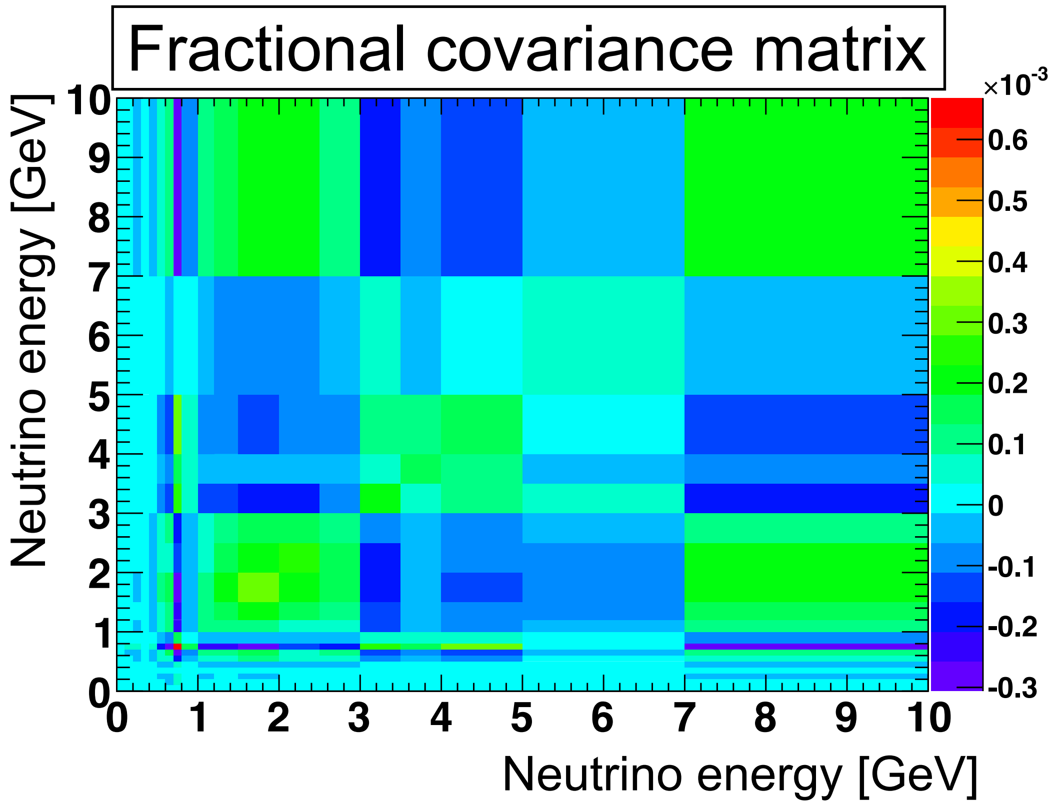
Flavor bins are 0 = ND5 nu_mu
1 = SK nu_mu
2 = SK nu_e

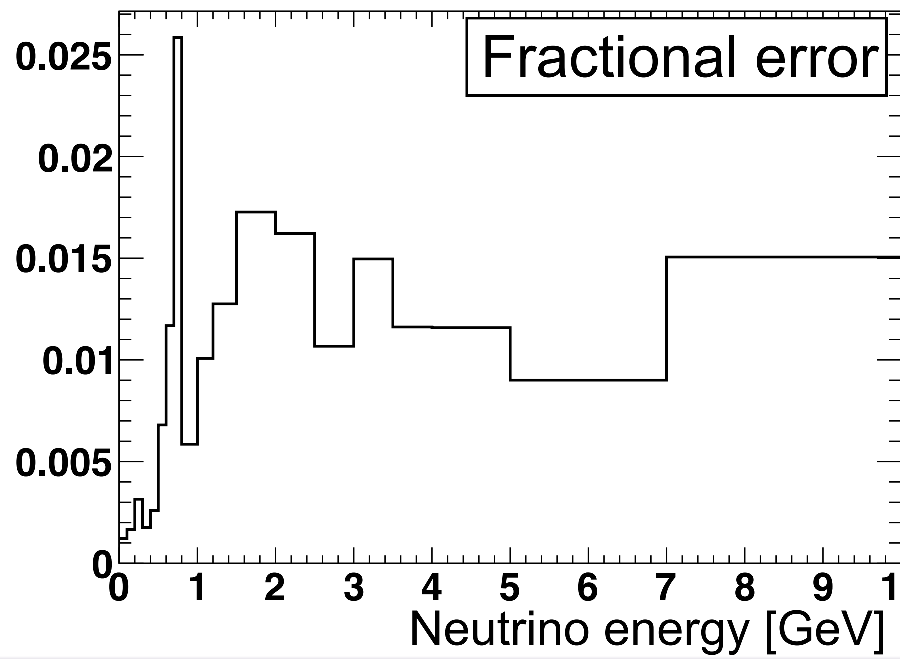
Proton Beam (F/N) SK:19bins / ND:9bins

Consider only Run1 (y-y' phase space)

Fractional error for SK, ND5 →

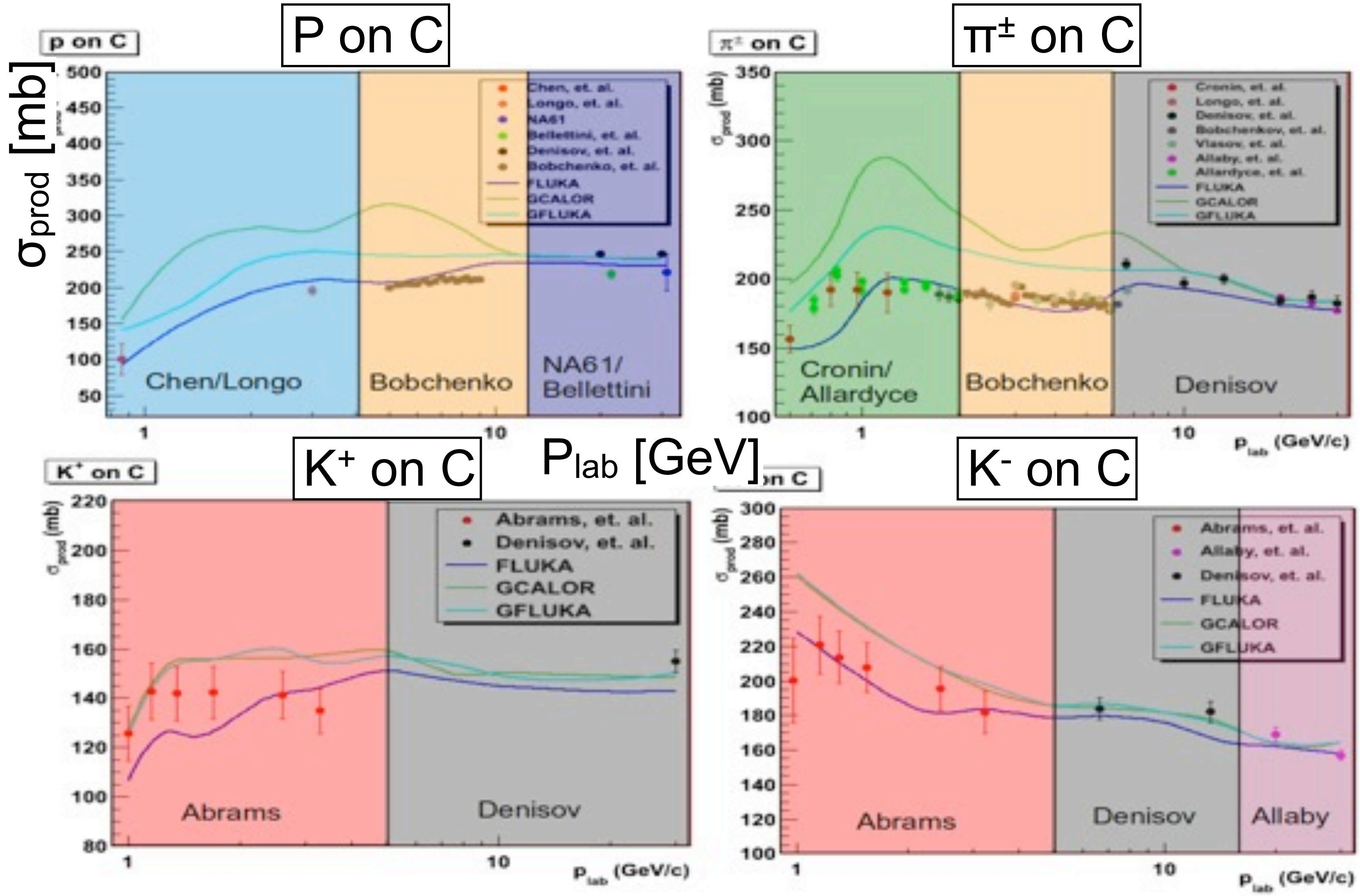






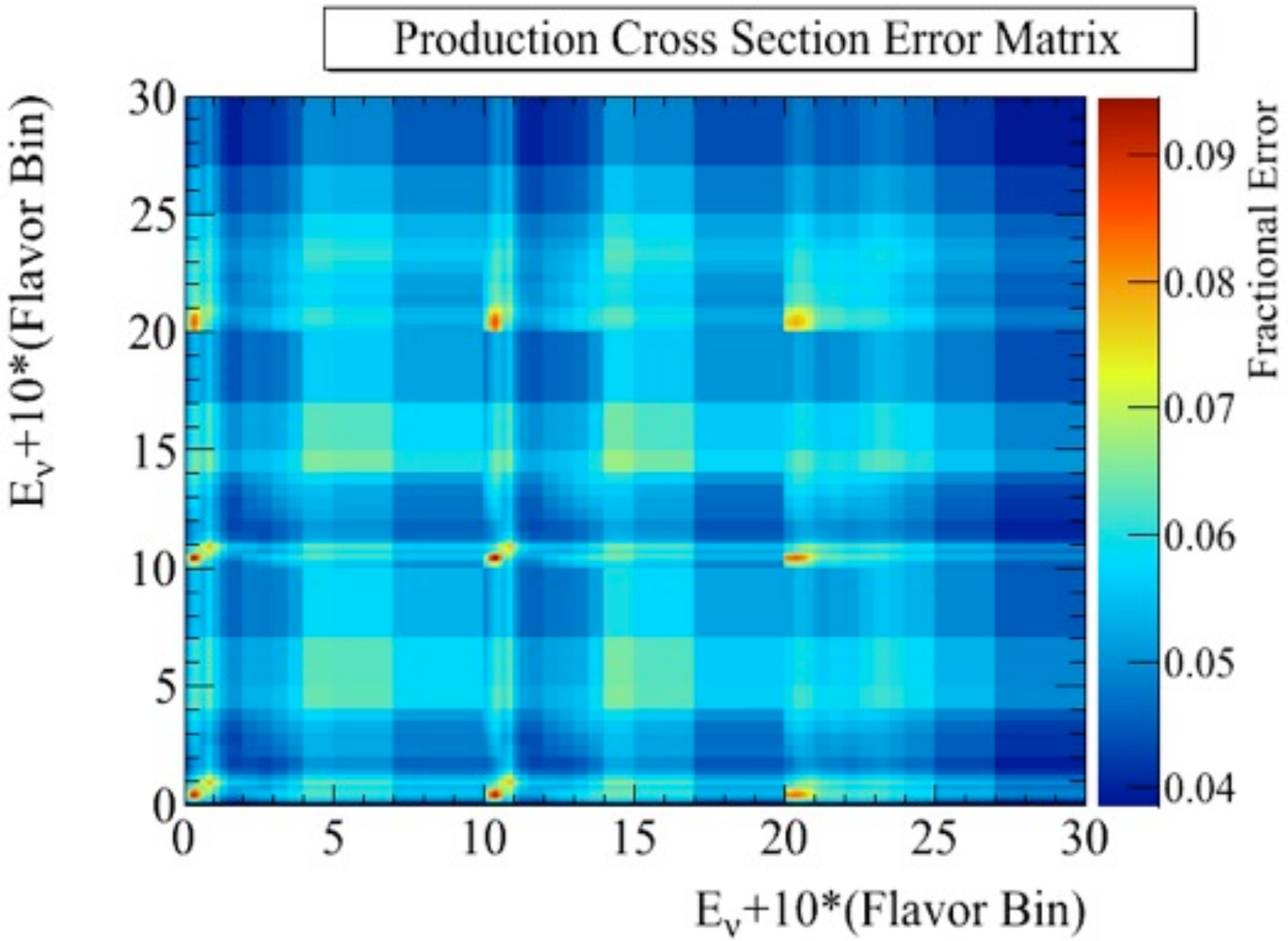
Production x-sec

- For cross section errors, we apply the quasi-elastic error independently in 6 different regions were different data sets apply
- Quasi-elastic error is equal to the size the calculated quasi-elastic component



Production x-sec (flux covariance)

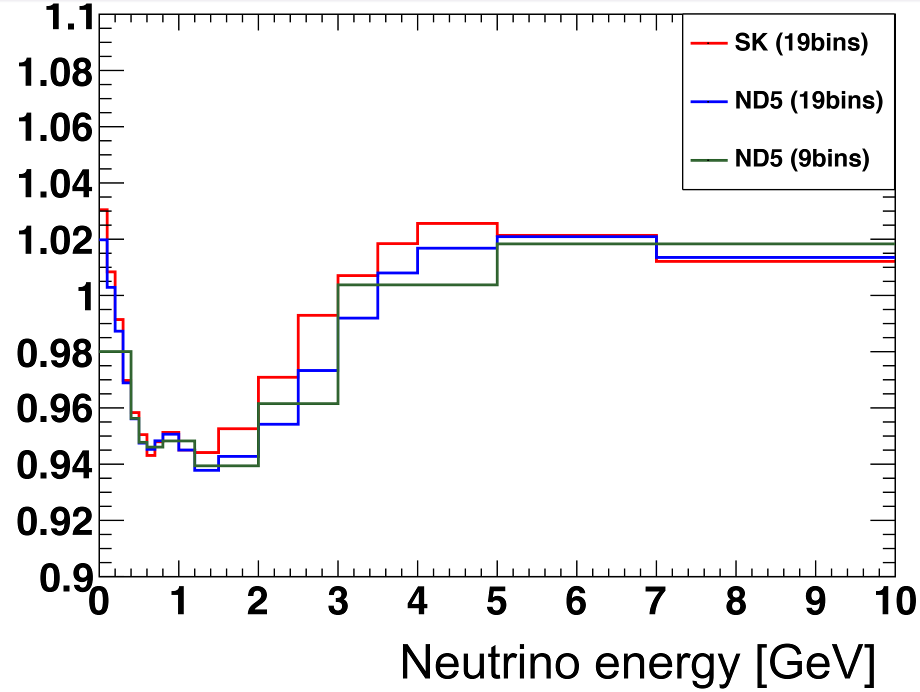
Production cross section error matrix:



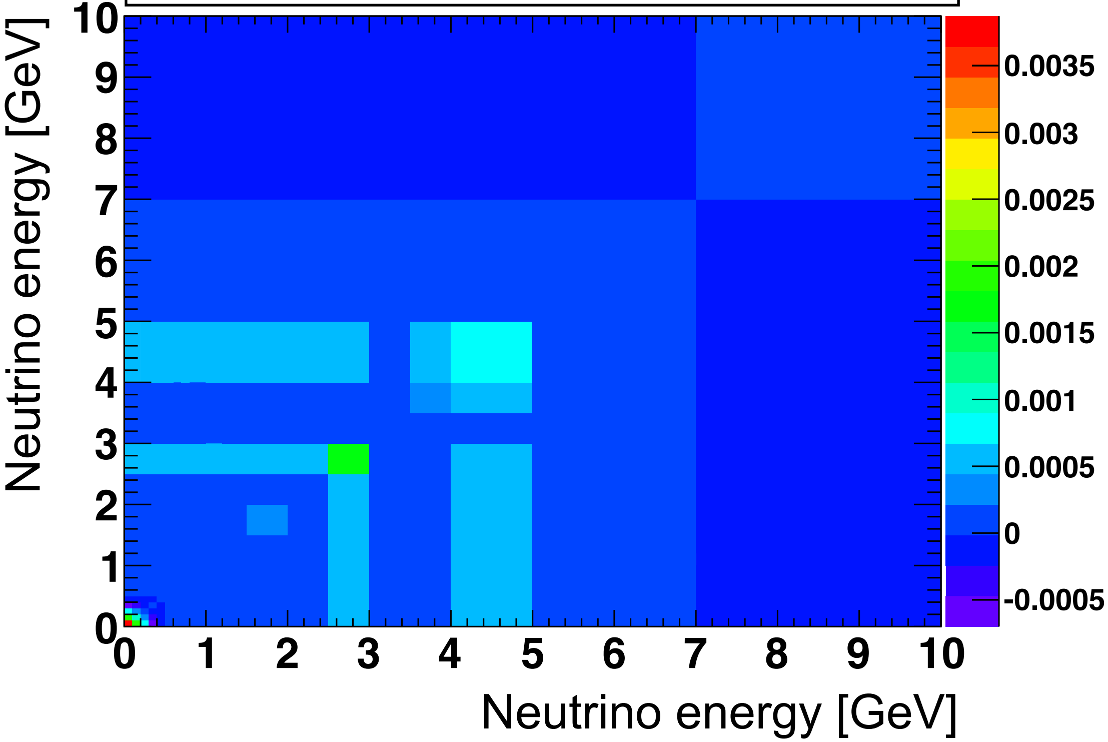
Flavor bins are 0 = ND5 nu_mu
1 = SK nu_mu
2 = SK nu_e

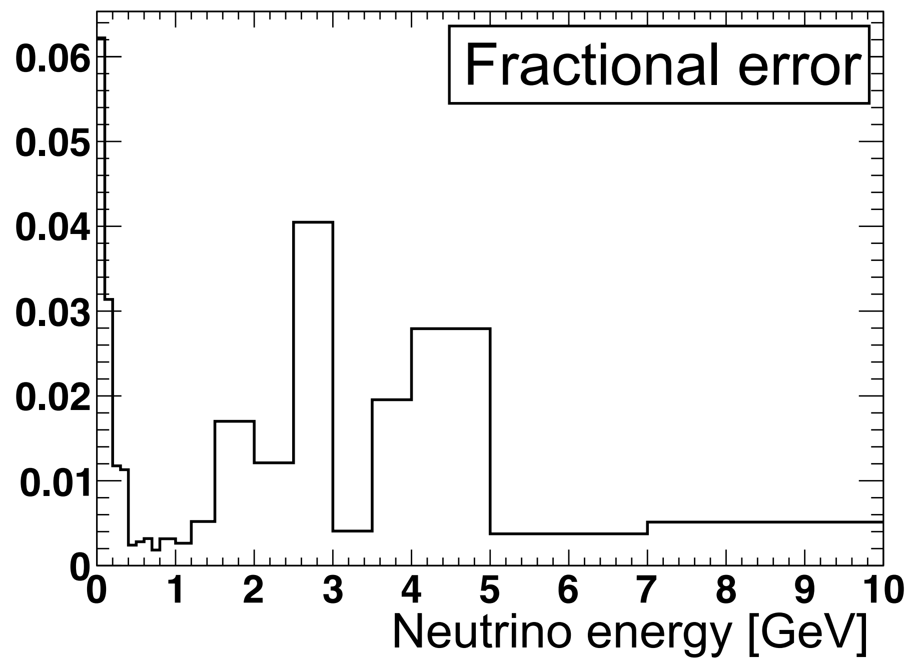
Production x-sec (F/N) SK:19bins / ND:9bins

Ratio of +1sigma changed flux to nominal one→



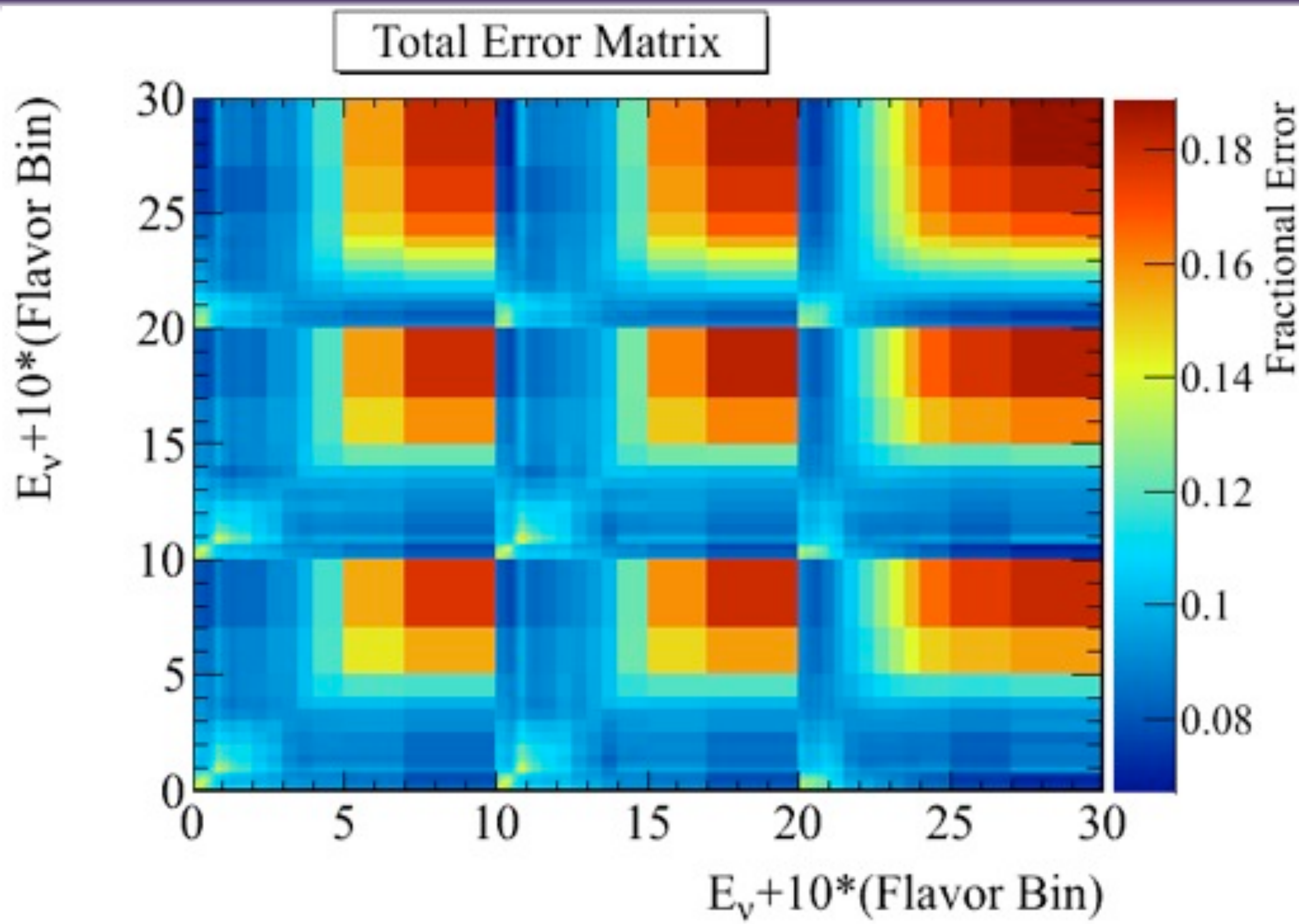
Fractional covariance matrix





Total Error Matrix (flux covariance)

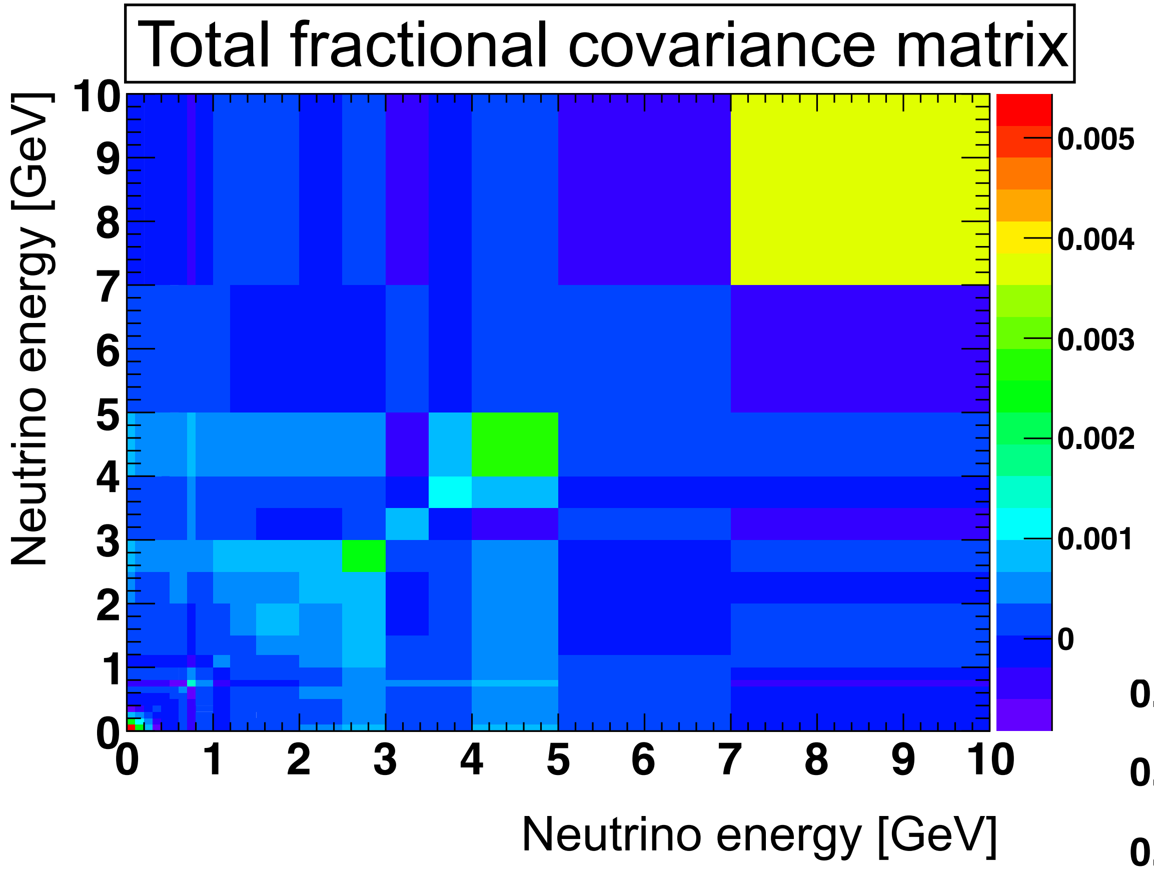
- Preliminary covariance is produced by summing over the contributions
- Need to add:
  - Horn & target alignment
  - Absolute horn current
  - Horn field asymmetry

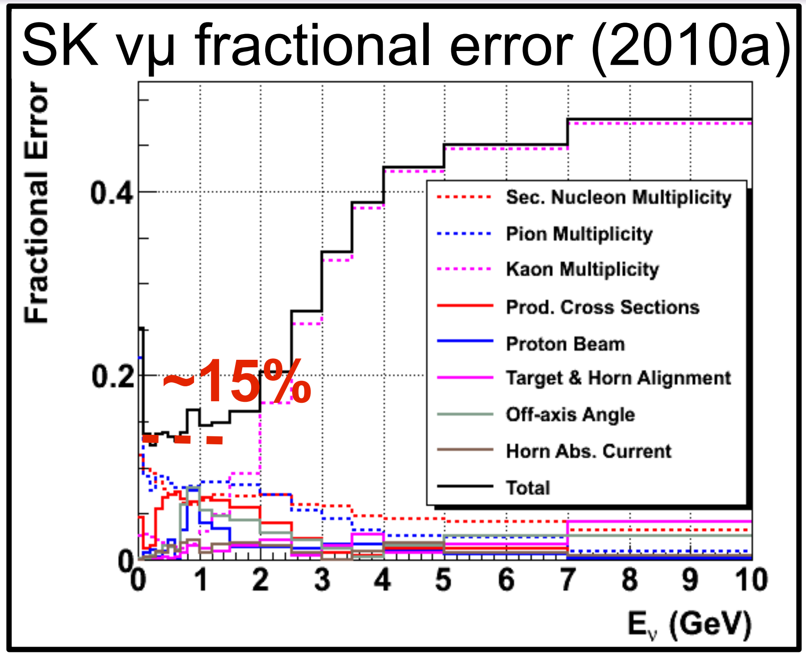


Flavor bins are 0 = ND5 nu_mu
1 = SK nu_mu
2 = SK nu_e

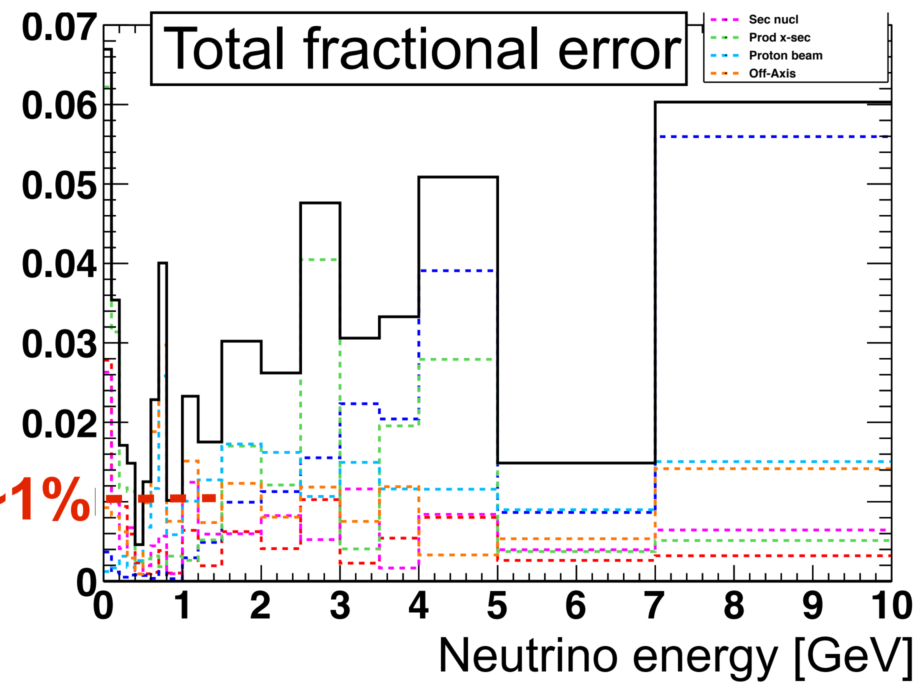
Significant off diagonal terms between ND5 νμ and SK νe

Total Error Matrix (F/N) SK:19bins / ND:9bins

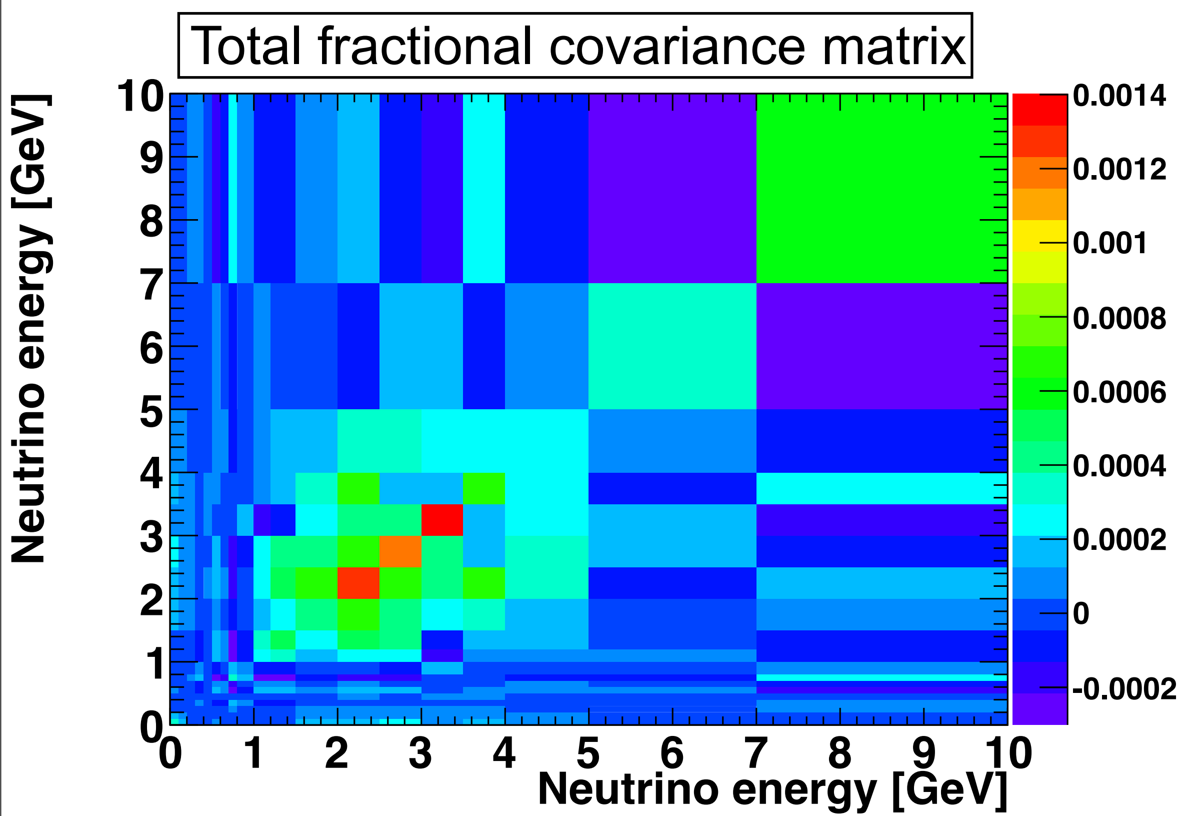


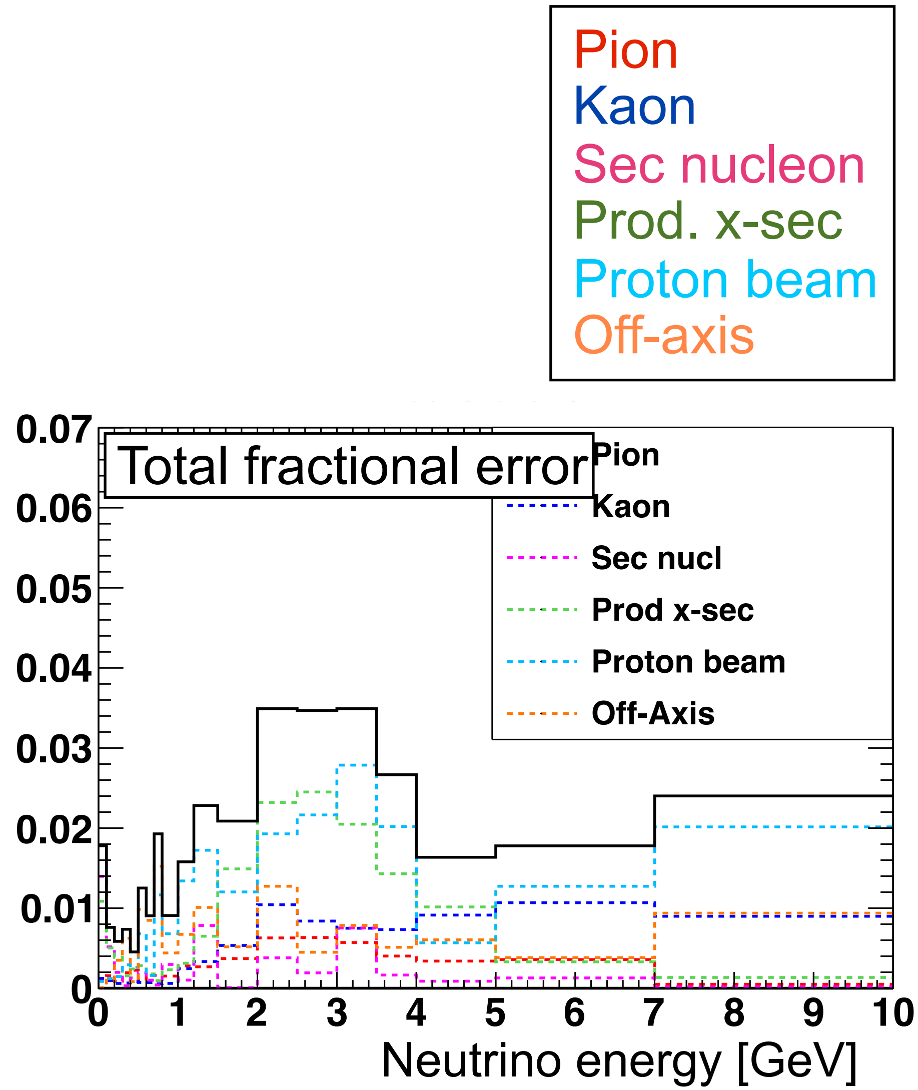


Pion
Kaon
Sec nucleon
Prod. x-sec
Proton beam
Off-axis



Total Error Matrix (F/N) SK:19bins / ND:**19bins**





Change bin combination: SK(19bins)/ ND(19bins)
(consider the case binning of ND is more fine).
→ Better cancellation between SK and ND

Input To 2011 Oscillation Analysis

For 2011 oscillation analysis, need flux covariance for ND280 nu_mu, SK nu_mu, SK nu_mu-bar and SK nu_e fluxes

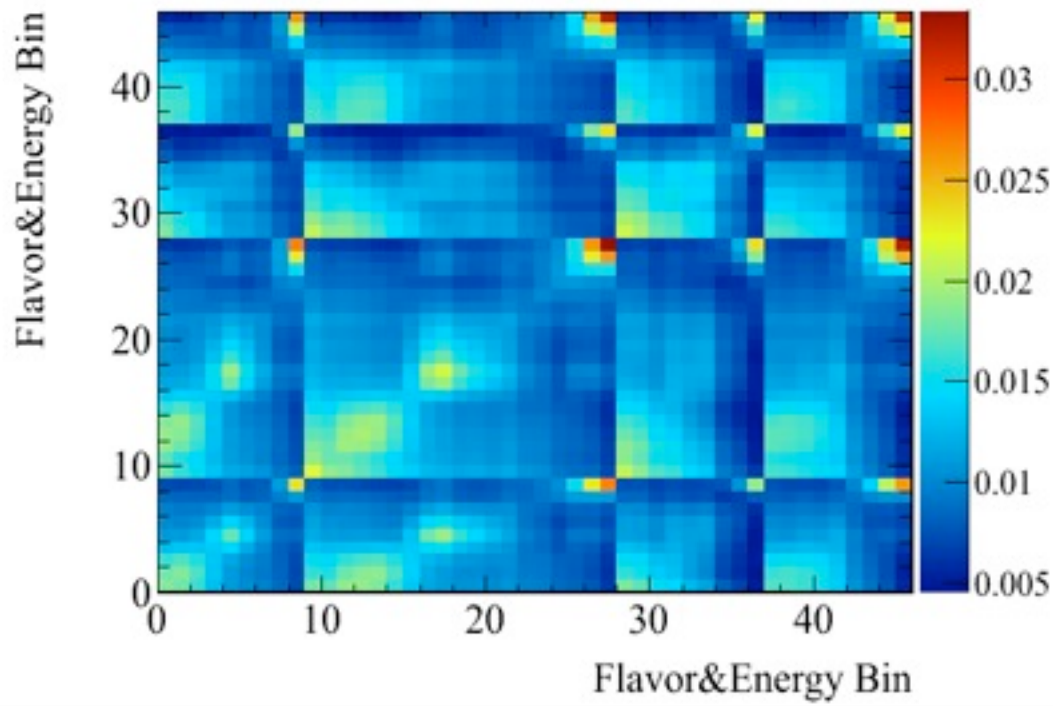
SK nu_mu binning:

```
int nbins = 19;
double bins[20] = {0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0,
                   1.2, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 5.0, 7.0, 10.0}
```

Others:

```
int nbins = 9;
double bins[10] = {0.0, 0.4, 0.5, 0.6, 0.8, 1.2, 2.0, 3.0, 5.0,10.0};
```

Unofficial covariance without most recent pion and kaon error updates is being used in 2011 oscillation analysis and BANFF studies

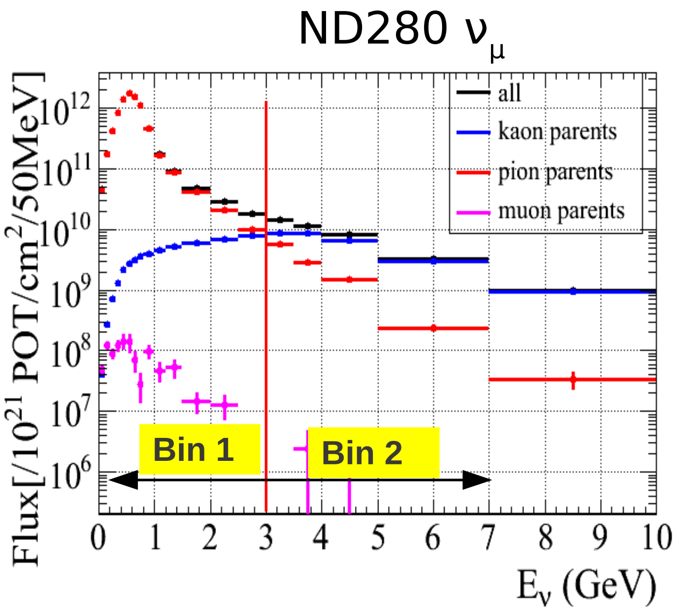


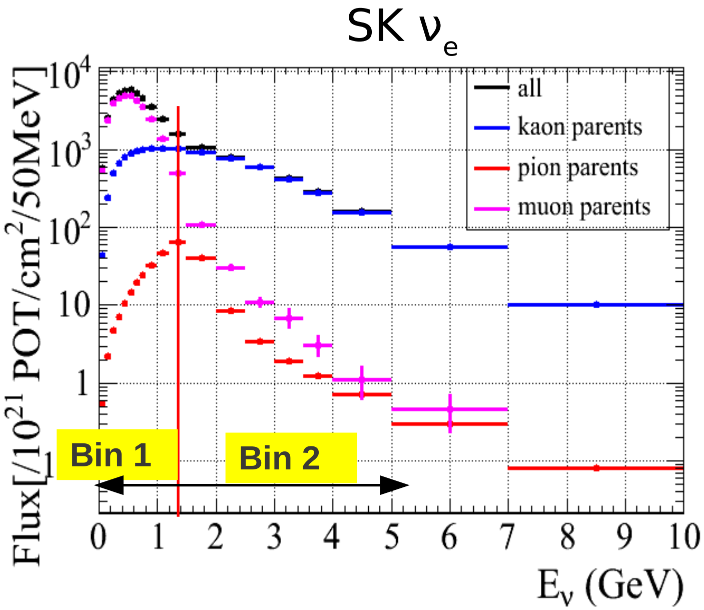
Summary & Things To Do

- Estimate the error of flux covariance and F/N ratio for each error source
- Estimation done about almost error source

- Need to consider other error source
  - Target and horn alignment errors
  - Horn absolute current error
  - Horn field asymmetry errors (when they are ready)
- Update according update of error estimation
  - Proton beam errors with wide beam studies
  - Secondary nucleon errors (when studies are complete)
- Make the error estimation for the flux covariance and F/N ratio consistent.
- Consider F(nue)/N(numu) extrapolation method (next page).

F(nue) / N(numu) study (halfway)





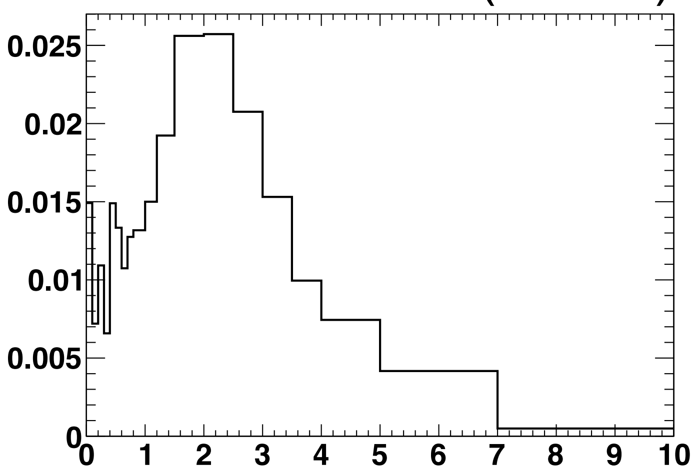
ν from pion
- ND280 νμ : <3GeV
- SK νe : <1.5GeV

Choose SK and ND280 bins to cancel flux error (F:νe / N:νμ)

Estimate F(nue)/N(numu) error with 19bins, 2bin example

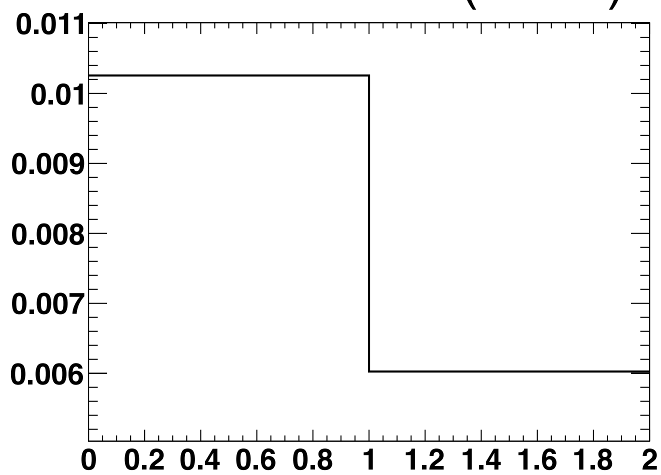
```
SK: 0~1.5, 1.5~10GeV [GeV]
ND280: 0~3.0, 3.0~10. [GeV]
```

Fractional error of F(19bins)/N(19bins)



Neutrino energy [GeV]

Fractional error of F(2bins)/N(2bins)



Energy bin#

Back up

Error propagation

V. Galymov

$$f_i^{(k)} - \bar{f}_i = \sum_{l=1}^{L} (x_l^{(k)} - \bar{x}_l) \frac{df_i}{dx_l} = \sum_{l=1}^{L} (x_l^{(k)} - \bar{x}_l) A_{il}$$

Content of i^{th} new bin in the k^{th} sample

Content of l^{th} old bin in the k^{th} sample

Transformation matrix. For linear sum (re-binning of histograms) only has 1.0 and 0.0

$$\langle (f_i - \bar{f}_i)(f_j - \bar{f}_j) \rangle = \sum_{l,m} A_{il} \langle (x_l - \bar{x}_l)(x_m - \bar{x}_m) \rangle A_{jm}$$

$$\mathbf{\Sigma^f = A \Sigma^x A^T}$$

If we can find the transformation matrix **A** that relates old to new binning we can compute the new covariance matrix from the old one

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Transformation matrix

V. Galymov

```
bins_edge[20] = {0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7,
0.8, 1.0, 1.2, 1.5, 2.0, 2.5,3.0, 3.5, 4.0, 5.0, 7.0, 10.0}
```

↓

```
bins_edge[11] = {0.0, 0.3, 0.4, 0.5, 0.6, 0.8, 1.2, 2.0,
3.0, 5.0, 10.0}
```

↓

$$\mathbf{A}_{10\times 19}$$

$$f_0 = x_0 + x_1 + x_2$$
$$f_1 = x_3$$

1st row: 1, 1, 1, 0, ..., 0
2nd row: 0, 0, 0, 1, 0, ..., 0
...

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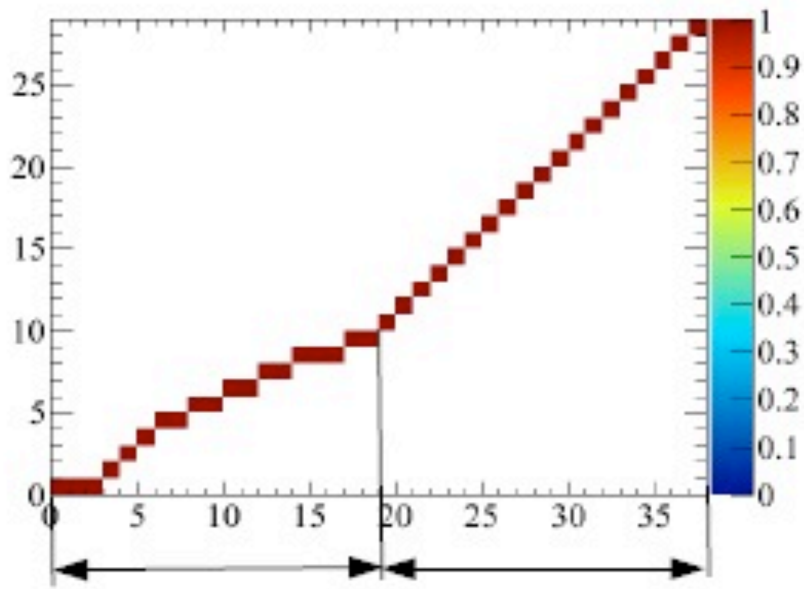
Some examples of binning transformations

V. Galymov

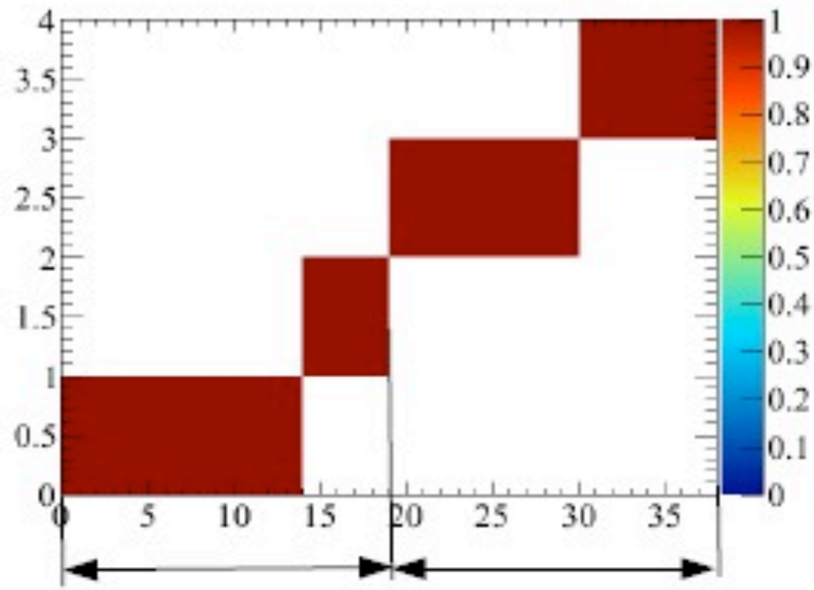
Original covariance matrix was produced using 19 bin for Enu for both ND280 and SK (covariance matrix is 38 x 38)

Transformation matrices for different binning scenarios:



ND5 numu 10 bins

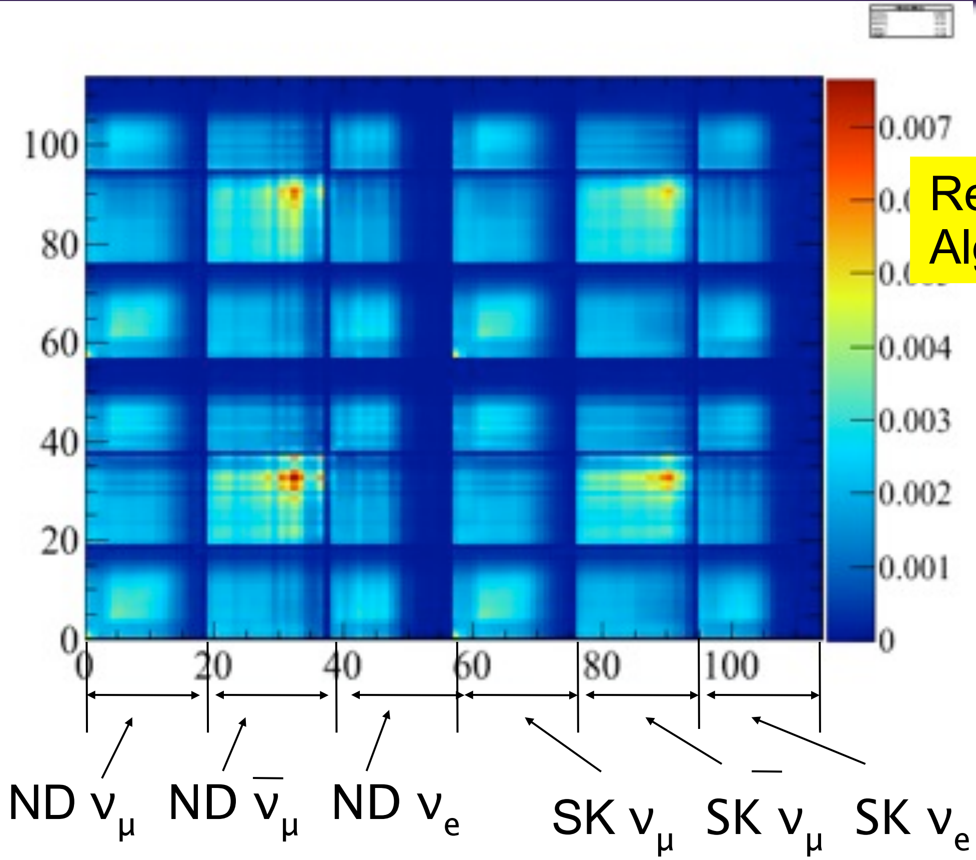
SK numu 19 bins



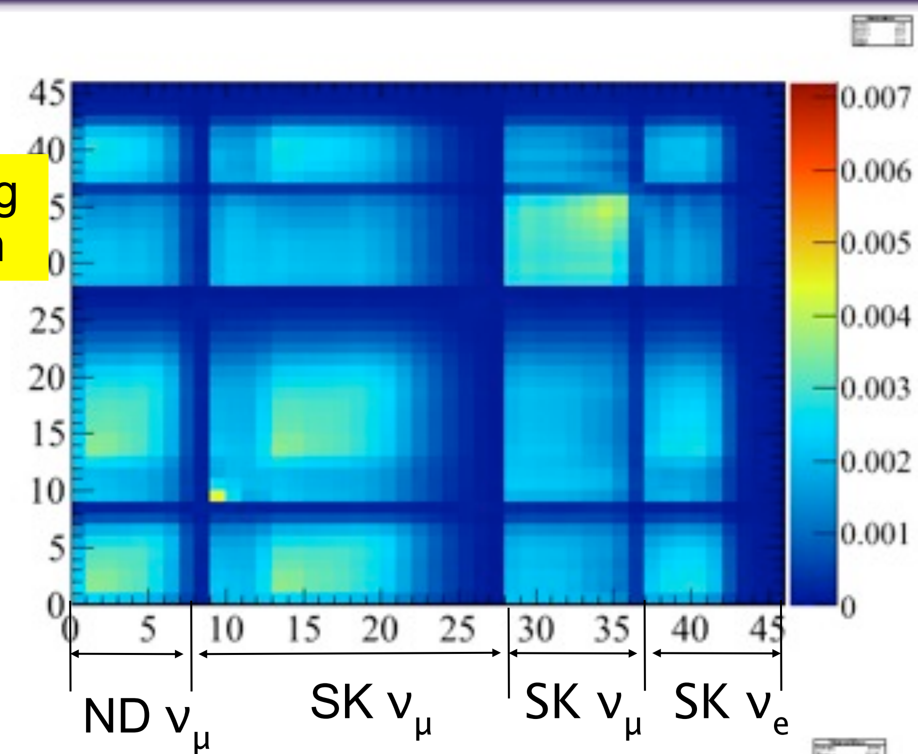
ND5 numu 2 bins:
1st: 0 – 3 GeV
2nd: 3 – 10 GeV

SK nue 2 bins:
1st: 0 – 1.5 GeV
2nd: 1.5 – 10 GeV

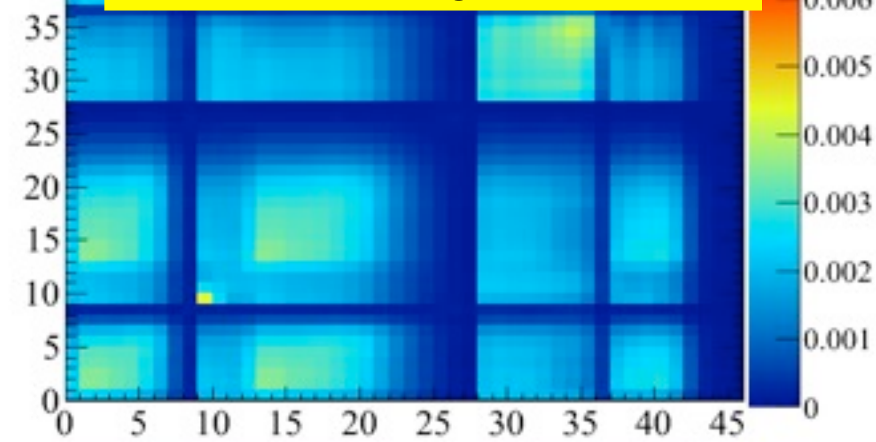
Rebinning Validation on Pion Covariance



Rebinning Algorithm



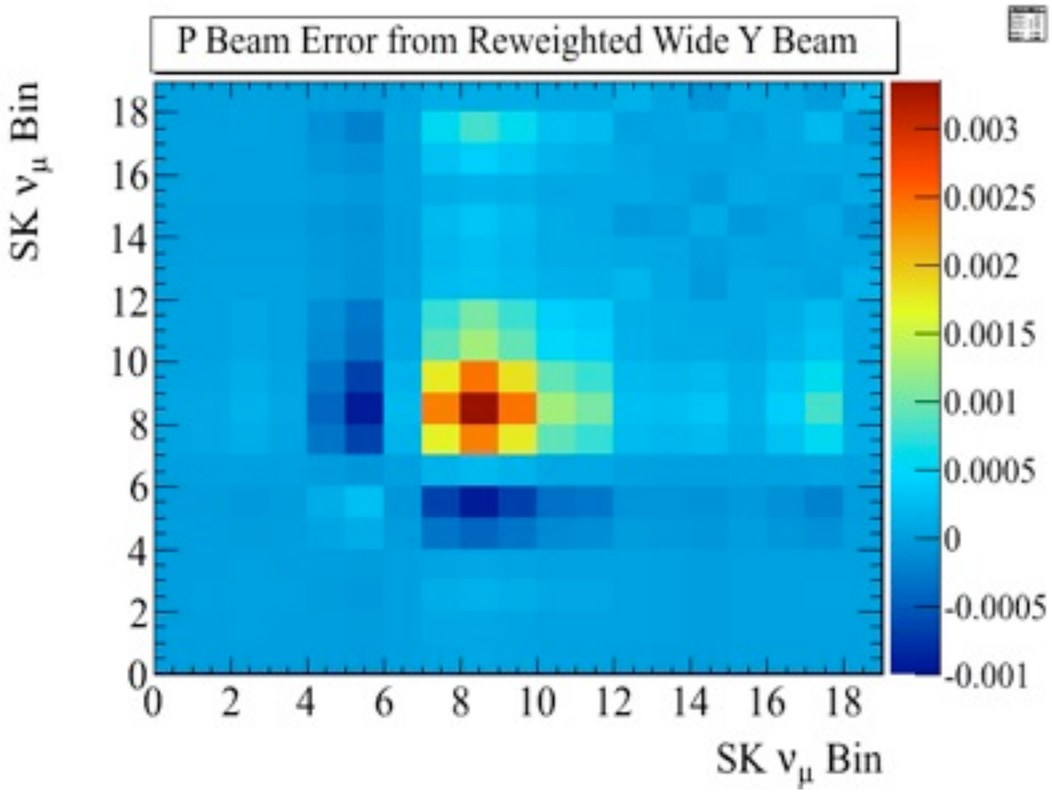
Covariance Calculate with modified binning



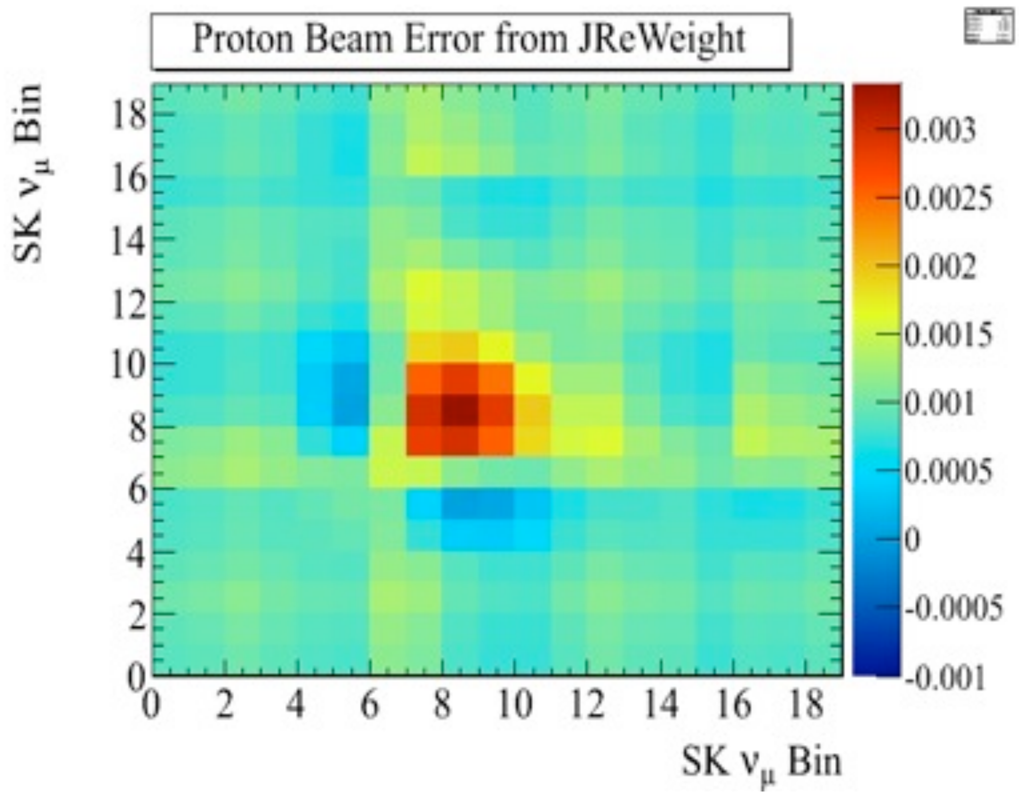
Covariance calculated with the rebinning algorithm is identical to that calculated with the modified binning from the beginning

Proton Beam

Reweighting of wide beam in Y only, with Run 1 errors. Used in 2010a



Reweighting with JReWeight (Use larger of Run 1 or Run 2 errors for each error source)



- Need to repeat reweighting with wide beam using Run 1 and Run 2 beam parameters.
- Can then be used to update covariance and validate JReWeight method