F/N extrapolate study

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- Check the correlation between ND280 flux and F/N
  - Make the covariance matrix including ND280 flux and F/N

F/N Ratio method (K2K style)

- Maximize ND280 likelihood **without the prior constrain of flux** → Get ND280 flux spectrum with covariance matrix.

$$L(\overrightarrow{\Phi}_{nd}, \overrightarrow{x}, \overrightarrow{n}) = \ln L_{xsec} + \ln L_{nd}(\overrightarrow{\Phi}_{nd}, \overrightarrow{x}, \overrightarrow{n})$$

- Construct SK flux prediction from got ND280 spectrum and F/N ratio

$$\Phi_{sk}(E_\nu) = \Phi_{nd}(E_\nu) f_{F/N}(E_\nu) \text{ (associated uncertainty } \mathrm{V}^{\mathrm{F/N}})$$

- SK likelihood depends on flux from ND280 and F/N ratio

$$L(\overrightarrow{o}, \overrightarrow{\Phi}_{nd}, \overrightarrow{f}_{F/N}, \overrightarrow{x}, \overrightarrow{s}) = \ln L_{xsec} + \ln L_{F/N} + \ln L_{nd} + \ln L_{sk}(\overrightarrow{\Phi}_{nd}, \overrightarrow{f}_{F/N}, \overrightarrow{o}, \overrightarrow{x}, \overrightarrow{s})$$

F/N Ratio method (T2K)

- In T2K, the ND280 flux is understood well from NA61 and so on.
- I think it is waste to do ND280 fitting without the well-known flux constrain → Include prior flux constrain in ND280 fitting.
- Changed ND280 likelihood including flux constrain:

$$L(\overrightarrow{\Phi}_{nd}, \overrightarrow{f}_{F/N}, \overrightarrow{x}, \overrightarrow{n}) = \ln L_{xsec} + \boxed{\ln L_{beam}(\overrightarrow{b}_{nd}, \overrightarrow{f}_{F/N})} + \ln L_{nd}(\overrightarrow{\Phi}_{nd}, \overrightarrow{x}, \overrightarrow{n})$$

→ new parameter and covariance (including F/N) in ND280 fitting

- And, SK likelihood:

$$L(\overrightarrow{o}, \overrightarrow{\Phi}_{nd}, \overrightarrow{f}_{F/N}, \overrightarrow{x}, \overrightarrow{s}) = \ln L_{xsec} + \ln L_{beam} + \ln L_{nd} + \ln L_{sk}$$

- Need the new beam covariance including the correlation between ND280 flux and F/N
  - If the correlation of ND280 and F/N is small, need not to consider this correlation.

Calculating covariance matrix

Basically same as one of F/N which I reported at last collabo.

- For independent errors sources, a total covariance can be derived by adding the covariance from each error source
- Two methods for calculating the covariance for a give error source
  - One sigma method : secondary nucleon, production x-section, OA angle
  - Throwing method : pion, kaon multiplicity, proton beam
- Estimated error size is same as my report

- ND280 bin(9bins) : bins[10] = {0.0, 0.4, 0.5, 0.6, 0.8, 1.2, 2.0, 3.0, 5.0,10.0} [GeV]
- SK bin(19bins) : {0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0,
1.2, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 5.0, 7.0, 10.0} [GeV]

F/N binning = SK(19bins) / ND280(9bin) (same as my report)

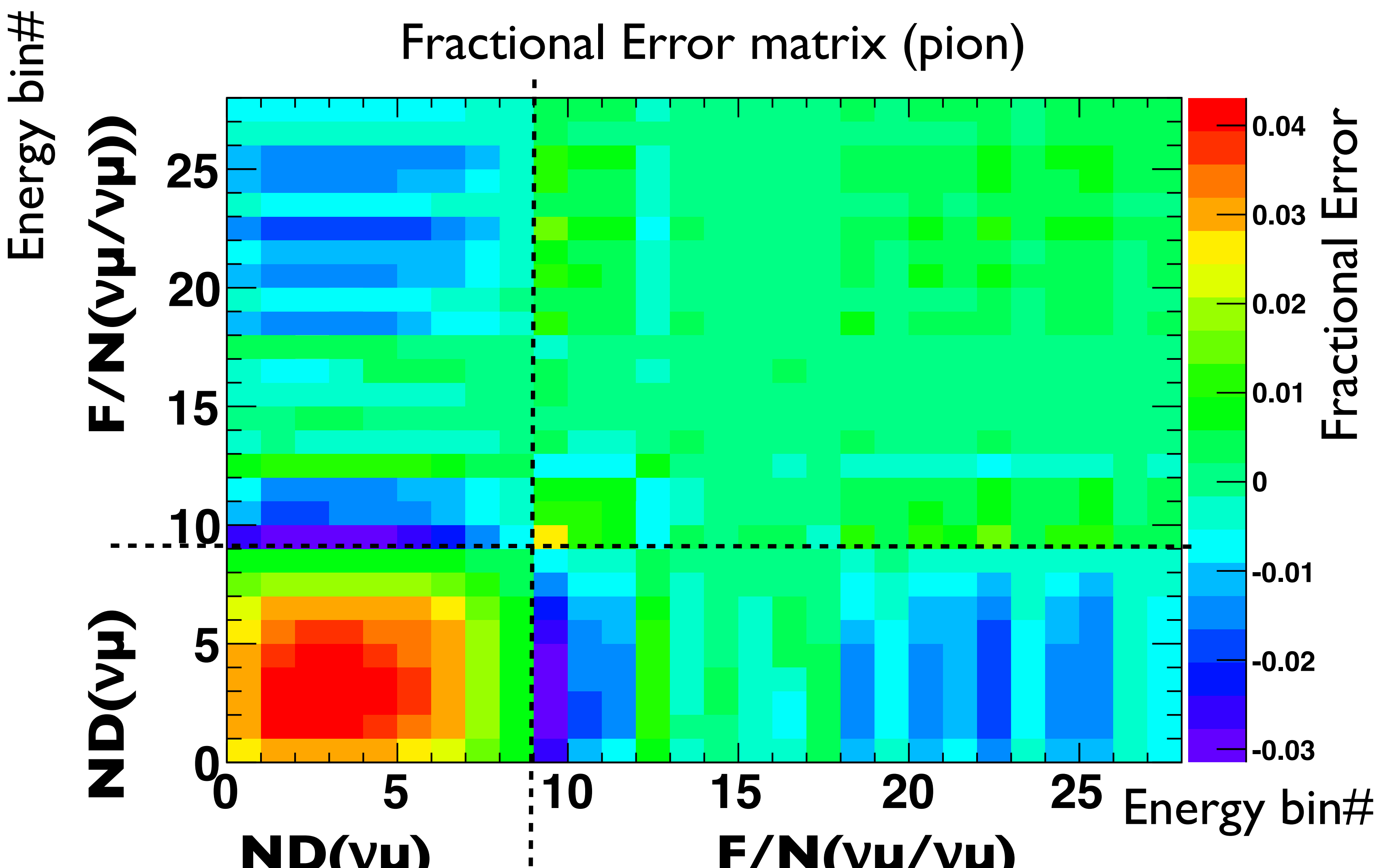
- **I show the error matrix instead of covariance matrix**

Error matrix:

$$E_{i,j} = sign(V_{i,j})\sqrt{|V_{i,j}|}$$

Pion error matrix

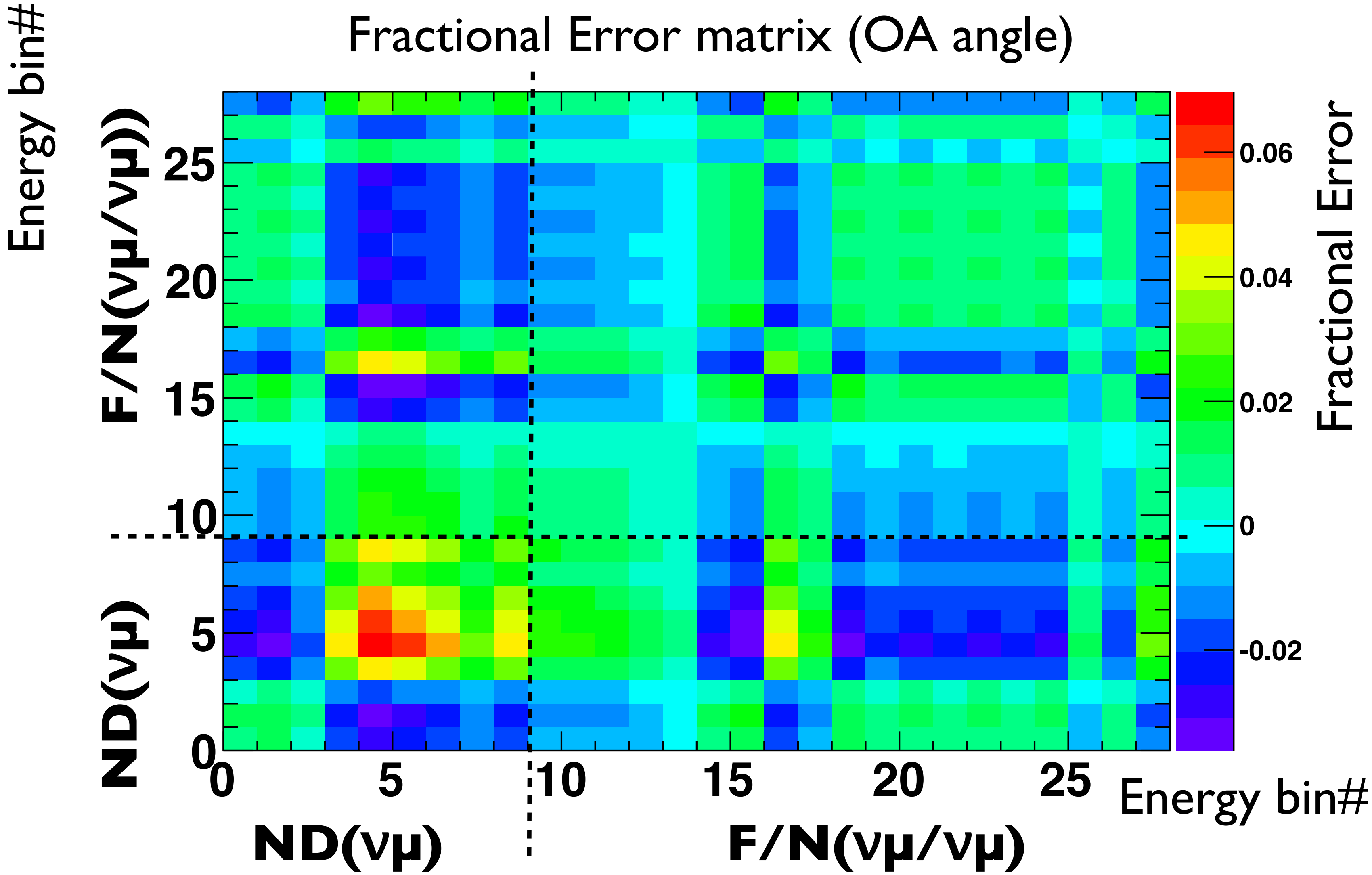
Throwing method



There is significant off-diagonal terms between ND and F/N .
The error size of ND-F/N is 1~2%, not negligible compared to the size of ND-ND terms(~4%).

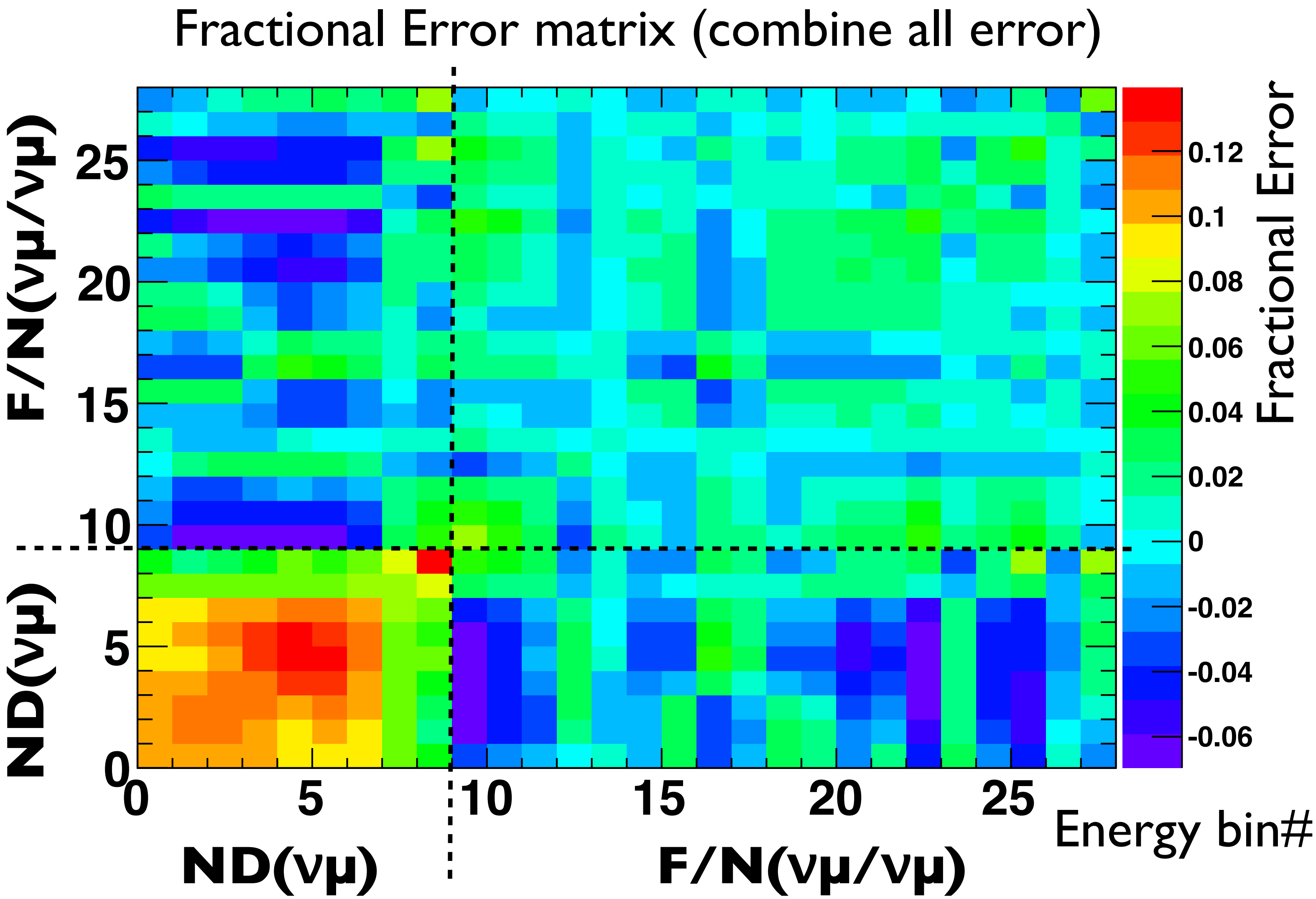
OA angle error matrix

One sigma method



Combined all error source

Combine all error source :
Pion multiplicity,
Kaon multiplicity,
Secondary nucleon,
Prod. x-section,
Proton beam,
OA angle



The error size of off-diagonal terms between ND and F/N is almost same as F/N-F/N terms, not negligible compared to ND-ND terms.

Summary

- There is non-negligible off-diagonal terms between ND $\nu\mu$ and F($\nu\mu$)/N($\nu\mu$).
  - But, for some error source, assume a certain correlation in one sigma method. Need to use throwing method to get precise covariance.
- Need to consider the covariance matrix including ND and F/N terms for nd280 fitting with prior flux constrain.
  - Need additional F/N fitting parameter in nd280 fitting... I think the main advantage of F/N method is the few fitting parameter. So, the advantage becomes less.
- How about SK νe? → Investigate F(νe)/N($\nu\mu$) 2bins method which I reported in last collabo.

Extra slide

Calculating covariance method

One sigma method:

- Fractional changes in the flux are calculated for +/- 1 sigma deviations of a single underlying parameter, e.g. off-axis angle
- Covariance is calculated from the fractional changes to the flux

$$V_{i,j}=\frac{\Delta\phi_i^{+1\sigma}\Delta\phi_j^{+1\sigma}+\Delta\phi_i^{-1\sigma}\Delta\phi_j^{-1\sigma}}{2}$$

Fractional change in flux (or F/N)

Throwing method:

- Many underlying parameters with their own covariance are varied at one time, e.g. NA61 pion multiplicity bins
- Underlying parameters are thrown many times according to their own covariance
- For each throw the flux is reweighted with the thrown parameter set
- The covariance is calculated from the reweighted flux in N throws

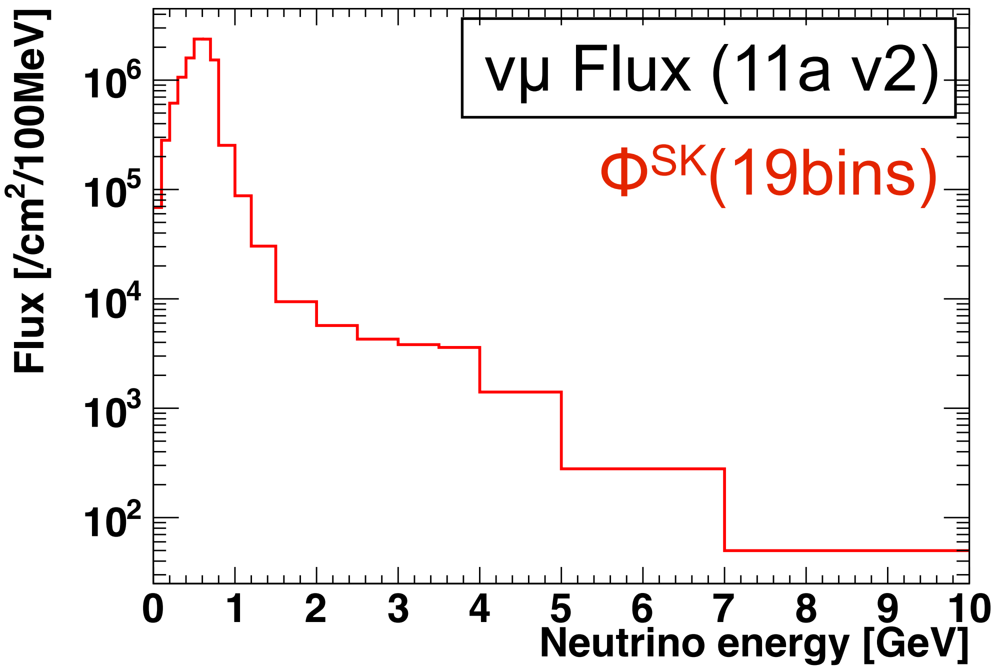
$$V_{i,j}=\frac{1}{N-1}\frac{\sum_{k=1}^{N}(\phi_i^{nom}-\phi_i^k)(\phi_j^{nom}-\phi_j^k)}{\phi_i^{nom}\phi_j^{nom}}$$

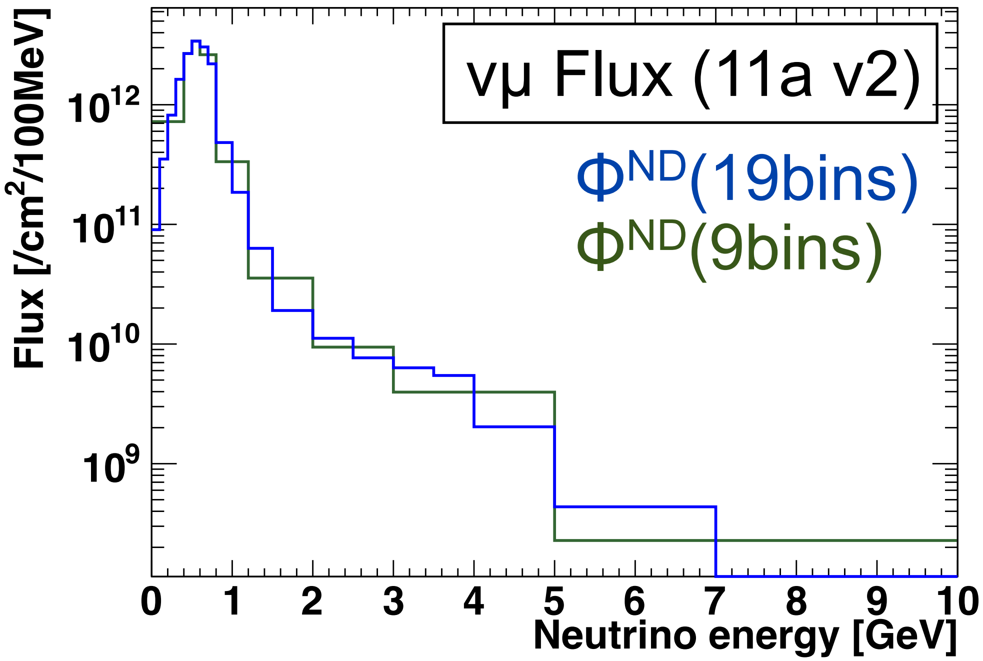
Error Sources for F/N

- ☑ **Pion multiplicity** – final 2007 NA61 pion result (not including Mark-san' update)
- ☑ **Kaon multiplicity** – updated with Slavic's report
- ☑ **Secondary nucleon** – from 1 sigma errors used in Run 1+2 analysis.
- ☑ **Off-axis angle** – from 1 sigma errors calculated for 0.44 mrad uncertainty
- ☑ **Proton beam** – estimated by throws using the reweighting based on proton beam uncertainty (only Run 1).
- ☑ **Production cross sections** – from throws of quasi-elastic uncertainty in different regions, same as 2010a nu_mu shape error method
- ☐ **Horn & target alignment** – from 1 sigma errors estimated for 2010a analysis, needs to be added
- ☐ **Absolute horn current** – from 1 sigma errors estimated for a 5 kA change, needs to be added
- ☐ **Horn field asymmetry** – based on results of field measurements, needs to be added

F/N Binning

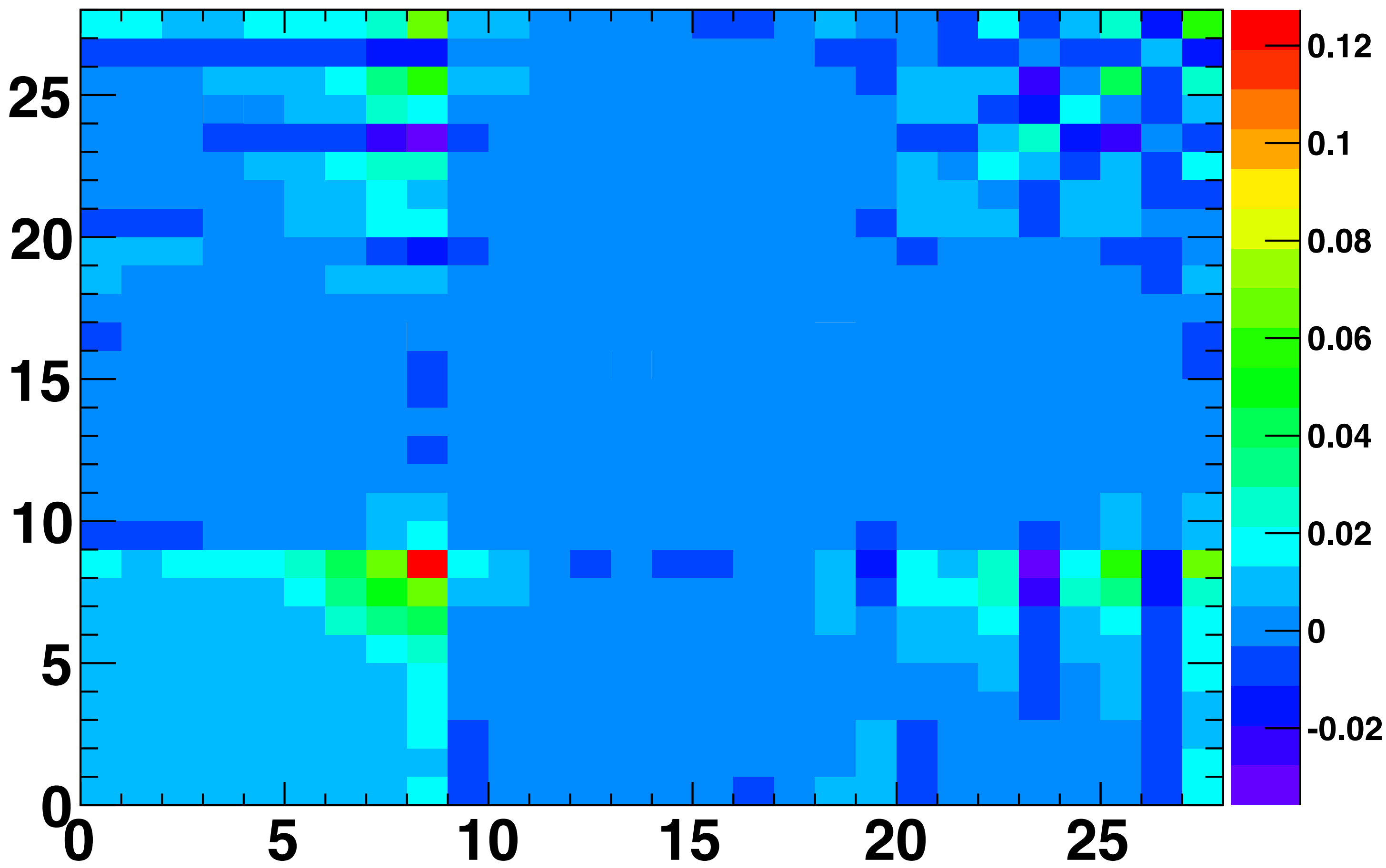
- Calculate F/N with the following binning to extrapolate the current ND280 fitting results to SK
  - SK(19bins) / ND(9bins)
    - Definitions of “19bins”, “9bins” are same as previous page.
- Calculate the flux ratio : the SK flux in each bin is divided by the ND5 flux in the bin which covers the SK bin.
  - $F/N(0\sim0.1GeV) = \Phi^{SK}(0\sim0.1GeV) / \Phi^{ND}(0\sim0.4GeV)$
  - $F/N(0.1\sim0.2GeV) = \Phi^{SK}(0.1\sim0.2GeV) / \Phi^{ND}(0\sim0.4GeV)$





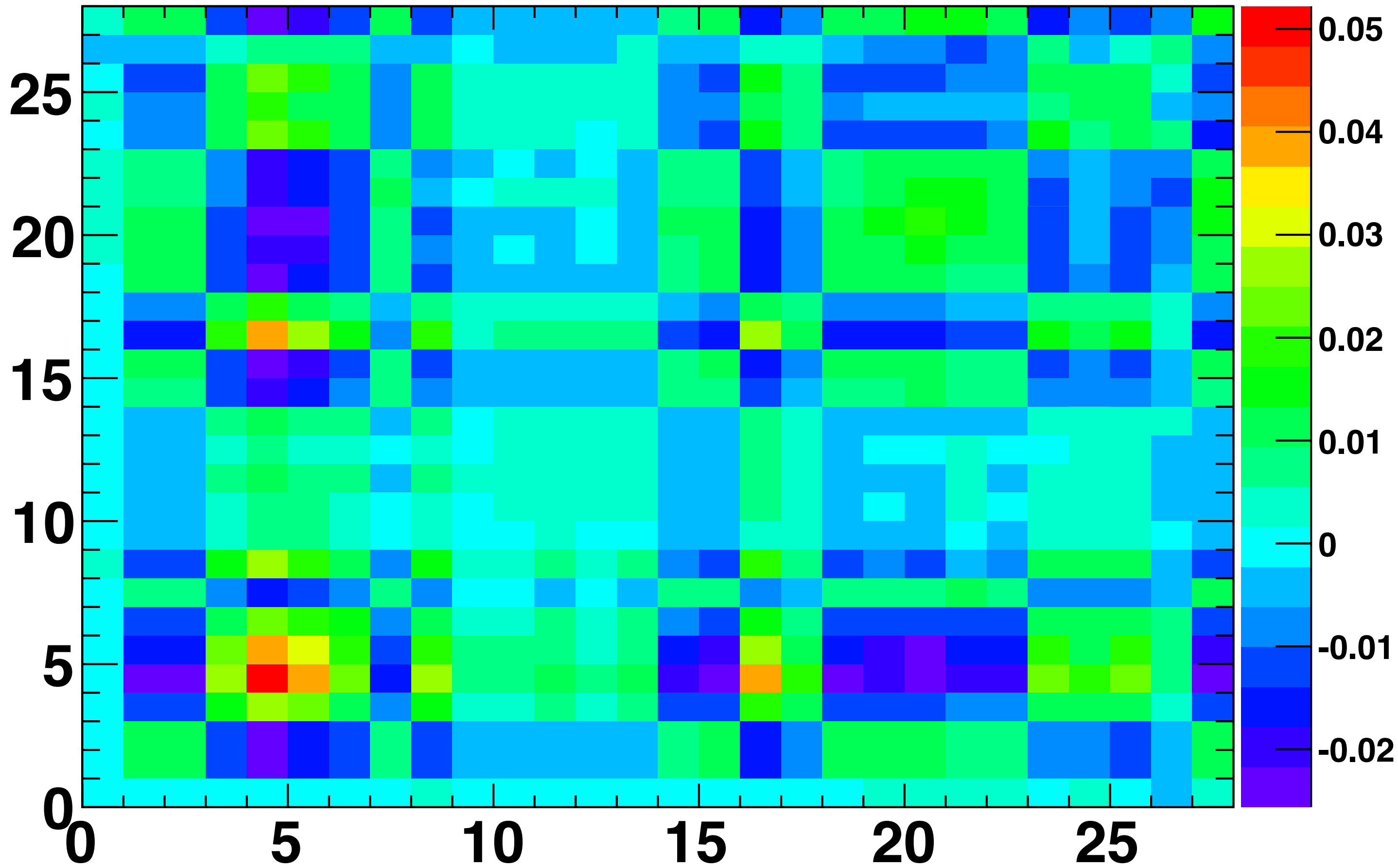
Kaon error matrix

Throwing method



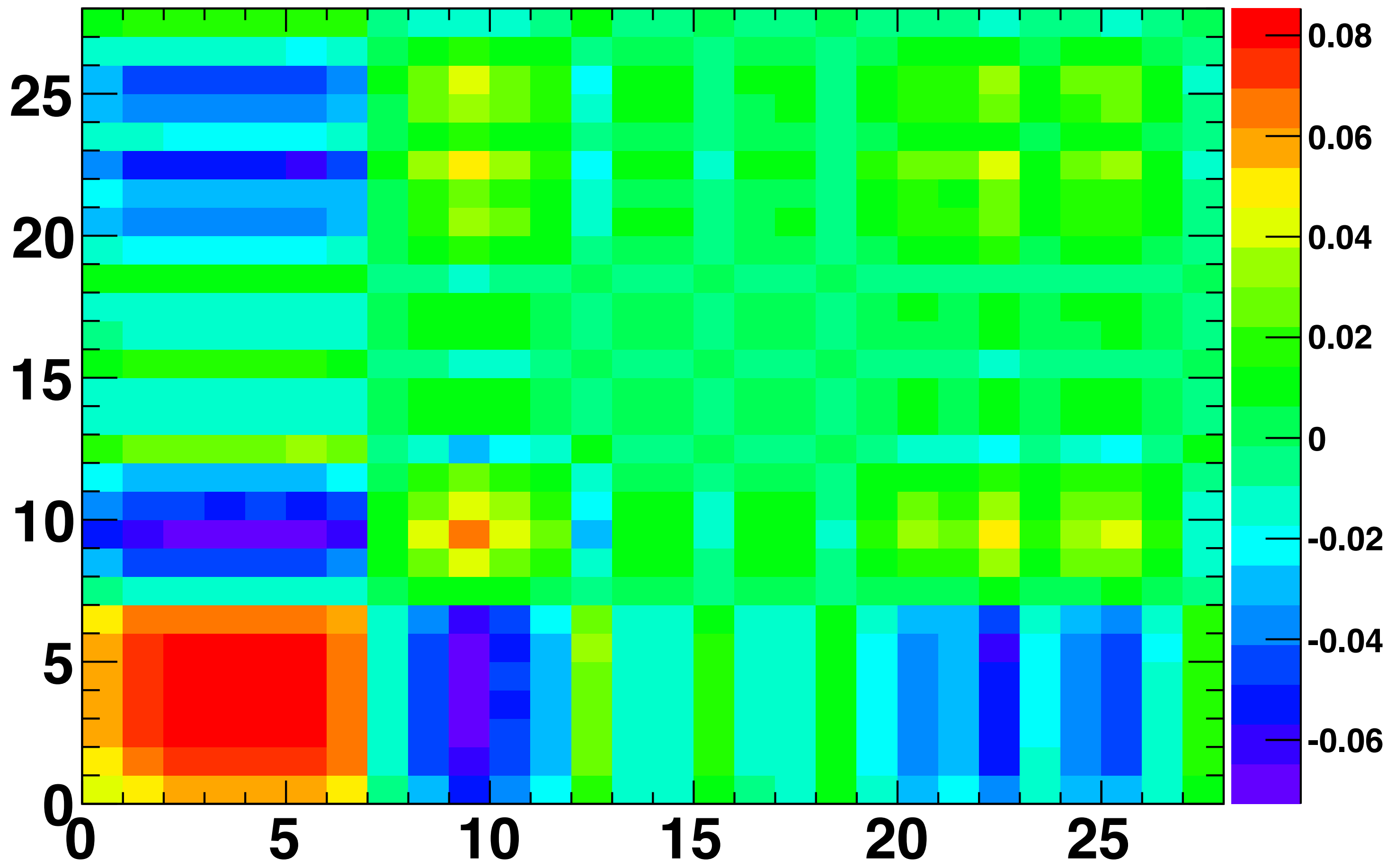
Proton beam

Throwing method



Prod. x-section

One sigma method



Secondary nucleon

One sigma method