INGRID Analysis Technical Note

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Abstract

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In this note we summarize the INGRID analysis results with 2010a data. We 8 measured the neutrino event rate, the beam profile center and these stability 9 for the confirmation and support of 2010a oscillation analysis. We select the 10 neutrino interaction, mainly charged current interaction, at each module and 11 reconstruct the neutrino beam profile. We compare some distributions between 12 data and MC and found good agreement. We get the data/MC ratio for the 13 event rate to be 1.072 ± 0.001 (stat.) ± 0.040 (syst.). The center of the neutrino 14 beam profile found to be $0.2 \pm 1.4(\text{stat.}) \pm 9.2(\text{syst.})$ cm for X profile and 15 -6.6 ± 1.5 (stat.) ± 10.4 (syst.) cm for Y profile. The neutrino beam direction 16 17 is measured as a direction from the beam origin to the center measured by INGRID and measured to be $-0.22 \pm .37$ mrad. 18

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³⁷ Chapter 1

Introduction

³⁹ INGRID is on-axis near detector which consists of identical 14 modules ¹ to
⁴⁰ monitor the beam stability. Each module has a sandwich structure made of iron
⁴¹ plates and scintillator trackers.

We count the number of neutrino interactions, mainly CC interaction, at each module. The neutrino event rate is monitored and the profile is reconstructed. Fig.1.1 shows a typical event in an INGRID module. Detector coordinates are shown in the figure. INGRID uses a right-handed Cartesian coordinate system in which the z axis is the beam direction and the y axis is the vertical upward direction.

48 This article shows the measurements of

49 (1) neutrino event rate and its stability

50 (2) neutrino beam profile center and its stability

Side View	Top View
-	140
	120
	100
-	80-
	60-
	40-
	20-

Figure 1.1: The typical neutrino event

This article is organized as follows. Chap.2 explains the overview of Monte Carlo simulation and Chap.3 describes the data set of 2010a. Chap.4 explains

 $^{^1\}mathrm{Additional}$ two off-center modules and a proton module are installed after 2010a data taking.

 $_{\tt 53}$ $\,$ the neutrino event selection. Finally the result of the event rate measurement

 $_{\rm 54}$ $\,$ and beam profile measurement are shown in Chap. 5 and Chap. 6, respectively.

⁵⁵ Chapter 2

Monte Carlo simulation

In this chapter, we explain Monte Carlo (MC) simulation used in this analysis.
 We use three MC simulation program : jnubeam, NEUT and detector sim-

- ⁵⁹ ulation (Fig.2.1).
- Neutrino flux : jubeam (version 10d tuned v2)
- Neutrino interaction to the target : NEUT (version 5.0.6.)
- Detector response simulation based on GEANT4¹

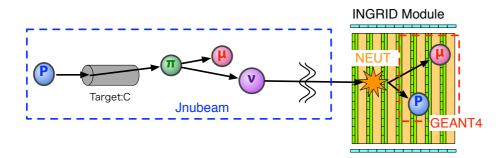


Figure 2.1: INGRID MC overview

⁶³ Currently, we simulate all the neutrino interaction with iron (Fe) as the tar ⁶⁴ get nuclei, although INGRID consists of iron, scintillator and support material.
 ⁶⁵ Simulation with correct material is under preparation.

We simulate the detector response to the generated particles from the neutrino interaction with simulator based GEANT4. We obtain the x and y position of the neutrino interaction from jubeam flux file. The vertex z is uniform in uni-

⁶⁹ formly generated in iron and scintillator taking into account the mass ratio of

¹This INGRID MC is not the software of ND280 software packages

⁷⁰ iron planes (99.54 ton) to scintillator planes (3.74 ton). The detector response
 ⁷¹ MC includes major effects which have an impact to the efficiency to neutrino
 ⁷² interaction :

- Quenching effect of scintillator and attenuation of photon propagating in the fiber.
- MPPC response model (including the effect of cross-talk and after pulse, and the effect of pixel saturation).
- Real geometry of scintillator bar which has effect on hit efficiency.

For MPPC response model, we refer to page 11 of the slide "Characterisa-78 tion of MPPC linearity response with the TRIP-T electronics " (reported by 79 Calibration group of ND280 working group in 2009. this slide put at t2k.org). 80 81 We tuned the conversion factor from energy to number of photon in MC with beam related sand muon by adjusting the peak p.e. by muon generated 82 in MC to that by sand muon in the real data. This tuning of scale factor is 83 accurate enough for current analysis, where the p.e. threshold is low enough 84 and is not critical to the efficiency to the neutrino events). 85

³⁶ Chapter 3

⁸⁷ Data set

- ⁸⁸ We took beam data during 2010 January to June. Data taking period, number
- ⁸⁹ of good spills, number of INGRID good spills are summazied int Table 3.1.
- $_{90}$ $\,$ Data taking efficiency for entire period is 99.9%, and total number of delivered
- 91 protons is 3.255×10^{19} .

MR run #	Period	Good spills	INGRID good spills	Protons at CT05
29	Jan. 23 - Feb. 5	26813	26813	0.32×10^{18}
30	Feb. 24 - Feb. 28	59256	59070	1.12×10^{18}
31	Mar. 19 - Mar. 25	86980	86935	$1.97 imes 10^{18}$
32	Arp. 14 - May. 1	237350	236647	7.64×10^{18}
33	May. 9 - Jun. 1	350079	350012	1.22×10^{19}
34	Jun. 7 - Jun. 26	246504	246410	9.30×10^{18}
Total		1006982	1005887	3.26×10^{19}

Table 3.1: Summary of datasets

³² Chapter 4

³³ Neutrino event selection

⁹⁴ 4.1 Event selection

We select the neutrino interaction to reconstruct the long track of charged particle started within fiducial volume of an INGRID module. Before reconstruction of the track, plane activity and photo-electron (PE) cut are applied to reject an accidental noise event. After reconstruction of the track, VETO cut and fiducial cut are applied to reject the incoming particle from the neutrino interaction at upstream materials. The order of event selections is shown below.

- 101 (1) Time clustering
- 102 (2) Number of active planes > 2
- 103 (3) PE/active layer > 6.5
- 104 (4) Tracking
- 105 (5) Track matching
- 106 (6) Beam timing cut
- 107 (7) Upstream VETO cut
- 108 (8) Fiducial volume cut

All selections are done a module by module and bunch by bunch basis. In this
analysis, the channel which has larger than 2.5 PE, which corresponds to TDC
threshold, is defined as the hit.

At first step hits are clustered with 100 nsec time window. Within the cluster the plane which has at least one coincidence hit in both x and y layers ¹, which is called active plane, is counted. Fig.4.1 shows the distribution of number of active planes and Fig.4.2 shows total PE divided by active layers. The event with less than 3 active planes or less than 6.5 PE is rejected.

¹INGRID module consists 11 planes and the plane consists 2 layers. Each layer has 24 scitillator bars and the direction of scitillator is perpendicular each other layer.

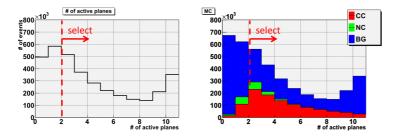


Figure 4.1: The number of active planes(left:DATA, right:MC normalized by area)

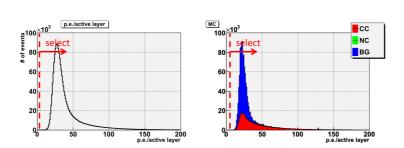


Figure 4.2: Total PE normalized by number of active layers(left:DATA, right:MC normalized by area)

After these selections track is reconstructed. First the hits in the most downstream active plane are adopted as end-point of the track. Looking at the hits in next upstream plane in order, the hit is adopted as track if calculated slope is matched with straight line.

After reconstruction of the track some badly fitted tracks are rejected by considering the between a 2-D track in x view and y view. Fig.4.4 shows the distribution of the difference of the vertex z between 2-D track in X-view and Y-view. We require the difference is smaller than 2 planes.

Because there are some background events such as cosmic-ray on beam off timing, the events within \pm 100 nsec from expected beam timing are selected (Fig.4.5).

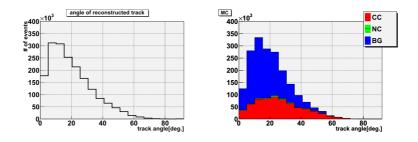


Figure 4.3: Angle of reconstructed track(left:DATA, right:MC normalized by area)

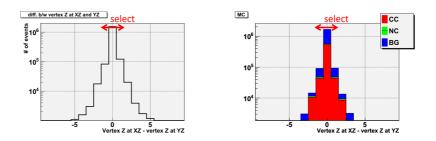


Figure 4.4: Difference of the vertex z in X-view and Y-view(left:DATA, right:MC normalized by area)

Finally we apply two selections to reject the incoming particles produced by the neutrino interactions in upstream materials. First one is upstream VETO selection. The track in which there is a hit in upstream position on VETO

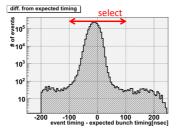


Figure 4.5: Time residual plot

 $_{131}$ $\,$ plane like Fig. 4.6 is rejected. After that we apply fiducial volume cut which is $_{132}$ $\,$ defined 100 \times 100 cm of each module (Fig4.7, Fig4.8 and 4.9).

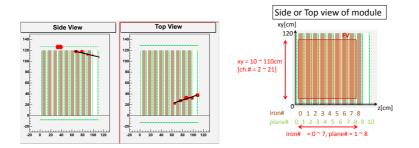


Figure 4.6: The event rejected by Figure 4.7: The definition of fiducial upstream VETO selection volume

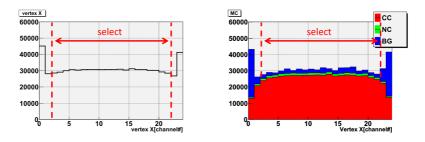


Figure 4.8: vertex x(left:DATA, right:MC normalized by area)

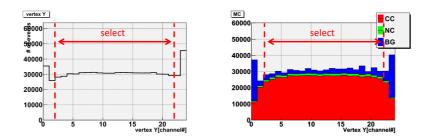


Figure 4.9: vertex y(left:DATA, right:MC normalized by area)

133 Event selection summary

	selection	Data		MC	
1	# of active planes > 2	1906146		$1.97 imes 10^6$	
2	PE / active layers > 6.5	1906078	(1.00)	$1.97 imes 10^6$	(1.00)
3	Tracking	1804786	(0.95)	$1.83 imes 10^6$	(0.93)
4	Track matching	1749548	(0.97)	1.77×10^6	(0.97)
5	Beam timing	1747181	(0.99)	1.77×10^6	(1.00)
6	Upstream VETO cut	745912	(0.43)	$7.35 imes 10^5$	(0.42)
7	Vertex in fiducial	493813	(0.66)	4.75×10^{5}	(0.66)

The number of events at each selection is summarized in Table 4.1. We obtained 493813 neutrino events during 2010a data taking.

Table 4.1: Summary of the event selection. Data and MC are normalized by pot

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¹³⁶ 4.2 Basic distribution of data and MC

In this section we show some distributions of selected events. In each distribution, there are two plots; one is overwriting of data and MC normalized by area
and one is data/MC. We found good agreement between DATA and MC.

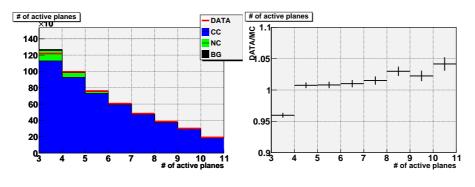


Figure 4.10: number of active planes

Figure 4.11: DATA/MC

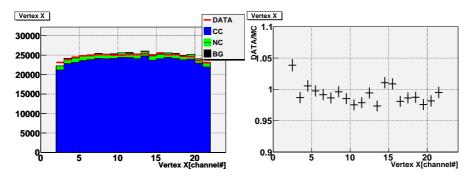
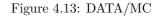


Figure 4.12: Vertex X



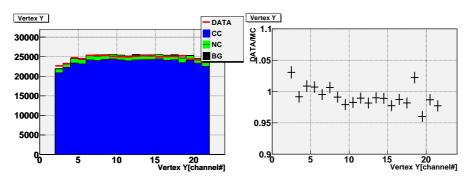


Figure 4.14: Vertex Y

Figure 4.15: DATA/MC

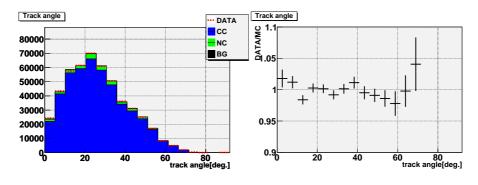
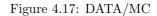


Figure 4.16: Reconstructed track angle



¹⁴⁰ 4.3 Reconstruction resolution

- 141 Reconstruction resolution is checked by MC to compare the reconstructed value
- $_{142}$ $\,$ and the MC true information. The results of vertex X, Y and track angle show
- Fig.4.18 and 4.19. Their r.m.s. for CCQE events are 2.7 cm for X, 2.8 cm for Y and 3.8 degree, respectively.

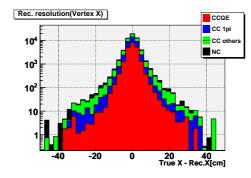


Figure 4.18: Reconstruction resolution of vertex X

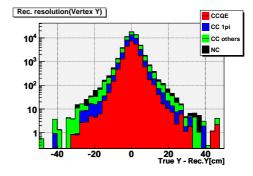


Figure 4.19: Reconstruction resolution of vertex Y

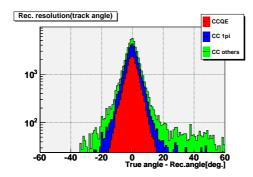
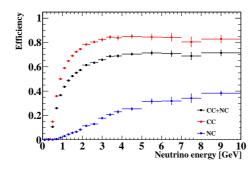


Figure 4.20: Reconstruction resolution of track angle

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¹⁴⁵ 4.4 Efficiency to neutrino interaction

The event selection efficiency is shown in Fig.4.21 and 4.22. Here, the efficiency is defined as the ratio of selected events to generated events within the fiducial volume. Table 4.2 shows mean of selection efficiency for each module. Because mean of neutrino energy is slightly different module by module, mean of efficiency is also different.



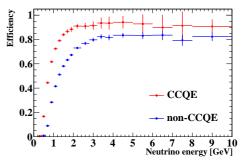


Figure 4.21: Neutrino event selection efficiency

Figure 4.22: Selection efficiency for CCQE and CC others.

module	Mean efficiency [%]
0	51.7
1	54.0
2	55.1
3	55.1
4	55.0
5	54.2
6	51.2
7	52.6
8	54.4
9	55.1
10	55.0
11	54.6
12	54.1
13	51.8

Table 4.2: Mean efficiency of each module

¹⁵¹ Chapter 5

152 Measurement of event rate

153 5.1 Event rate stability

Fig.5.1 shows daily event rate normalized by delivered pot. We succeeded to
measure the daily event rate with about 1.7% statistical error each day. The chisquare calculated from the average rate is almost one to the degree of freedom.
It is concluded that the beam events is stable within statistical error.

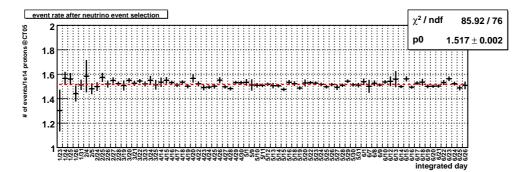


Figure 5.1: The stability of daily event rate

$_{158}$ 5.2 data/MC of event rate

To obtain the number of events in the fiducial volume $(N^{obs.})$, we need to do following corrections.

- ¹⁶¹ (1) Accidental MPPC noise
- 162 (2) Iron mass

(3) Beam related background 163

Detail description is in Chap. A. 164

- We derive the formula to evaluate the number of events in the fiducial volume from number of selected events (N^{sel.}); N^{obs.} = N^{sel.} × $\frac{1}{1+C}$, where C is the correction factor. The corrected number of events are summarized in Table 165
- 166
- 167

5.1 respectively. Detail for it is summarized in Chap. A. Finally we obtain 168 DATA/MC to be 1.072 ± 0.001 (stat.).

> Number of selected events 493813 Corrected number of events 508511

Table 5.1: Number of events before and after corrections

169

¹⁷⁰ 5.3 Systematic error of event rate

¹⁷¹ Table 5.2 shows the systematic errors which does not include physics uncertainty such as neutrino cross-section.

Item	Error ^[%]
Accidental MPPC noise	0.7
Iron mass	0.1
Beam related background	0.2
Fiducial selection	1.1
Hit efficiency	1.8
Tracking efficiency	1.4
Track matching selection	2.7
Not beam-related background	< 0.1
p.e./active layer selection	< 0.1
Beam timing selection	< 0.1
Total	3.7

Table 5.2: Systematic error table

172

173 Accidental MPPC noise

The effect of MPPC noise is studied with MC in which the MPPC noise hit 174 is generated to reproduce number of PE, timing, and noise rate of DATA. We 175 found that the more MPPC noise rate is, the more neutrino events are lost due 176 to miss identification of vertex Z or miss counting of number of active planes. 177 Its effect is found to be linear and slope is estimated to be -0.9585 %. Two 178 sources of systematic errors are considered. First one comes from the error 179 on the linear fit. To get this systematic error, we multiply the fit error by the 180 maximal measured noise rate. Second one comes from the measurement of noise. 181 Correction factors are calculated using the average noise rate measured on one 182 period. But this noise rate fluctuates in time (probably due to temperature 183 variations). So we measure the maximal difference between average noise rate 184 and noise rate measured at different times during one period, and using the 185 186 linear relation between noise rate and variation of number of events we get the systematic error. The quadratic sum of these two errors is 0.7 %. 187

188 Iron mass

Before construction of INGRID the mass of each iron plate was measured with a precision of 1 kg, which corresponds to 0.13 % of the mass of one iron plate. We will use this figure as the systematic error on this correction factor. We might need to increase this systematic error in the future, as the correction factors are calculated using the mass of the whole iron plate, when we actually use a set of the whole iron plate. We make a set of the whole iron plate is a set of the whole iron plate. The set of the set of the set of the whole iron plate.

fiducial cut in analysis, only interactions in the central part of the iron plates are kept.

¹⁹⁶ Beam-related background

We estimated the contamination fraction of beam related background with wall 197 neutrino Monte Carlo. The fraction is estimated to be 0.4%, in which the 198 number of interactions of background is normalized to compare the number of 199 dirt muon in DATA and MC. There is a 35% difference from POT expectation, 200 which is considered as one of the source of the systematic error. We considered 201 20% neutrino flux uncertainty and 20% cross section uncertainty as other sources 202 of the systematic error. Finally 0.2% $(=\sqrt{0.35^2+0.2^2+0.2^2})$ is applied as the 203 systematic error. 204

205 Fiducial selection

²⁰⁶ To estimate the uncertainty of fiducial selection and the effect of non uniformity

of iron plate, we divided fiducial in several horizontal slices and checked the
 difference between DATA and MC. Table 5.3 shows the result. The maximum absolute value, 1.1%, is applied as systematic error.

selection	DATA	MC	DATA - MC
<50 cm from center(nominal selection)	100.0	100.0	0.0
$<\!25 \mathrm{~cm}$	25.6	25.2	0.4
$25 \sim 40 \text{ cm}$	39.9	39.3	0.6
$40 \sim 50 \text{ cm}$	34.4	35.5	1.1
Systematic error (Maximum absolute)			1.1

Table 5.3: DATA-MC for several sub fiducial volume

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210 Hit efficiency

We estimated the relation between hit efficiency and number of selected events with MC. Fig. 5.2 shows the result from which the systematic error of hit efficiency is estimated to be 1.8% because hit efficiency has 1.1% uncertainty.¹

215 Track matching selection

In the neutrino event selection, after reconstruction of XZ track and YZ track we
require track start point matching. To estimate the uncertainty of the selection,
we changed the tolerance for the matching and checked the difference of the
number of selected events between DATA and MC. Table 5.4 shows the result.
The maximum absolute value, 2.7%, is applied as systematic error.

 $^{^{-1}0.5\%}$ of the measurement error of hit efficiency, 1.0% of the tuning of hit efficiency in MC

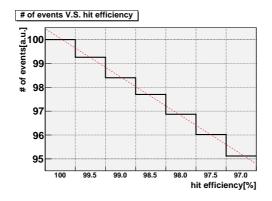


Figure 5.2: hit efficiency V.S. number of selected events

Vertex Z of XZ track - Vertex Z of YZ track	DATA	MC	DATA - MC
-1, 0, +1 (nominal selection)	100.0	100.0	0.0
0	83.0	85.7	2.7
-2, -1, 0, +1, +2	104.0	103.0	1.0
Systematic error (Maximum)			2.7

Table 5.4: DATA-MC for several tolerance of track matching.

221 Tracking efficiency

To check the difference of the tracking efficiency between DATA and MC, the tracking efficiency is compared with several sub-sample selected by number of active planes. Table 5.5 shows the result. The maximum absolute value, 1.4%, is applied as systematic error.

226 Not beam-related background

The off-bunch data (cycle $17 \sim 22$ where as on-bunch cycle is $4 \sim 9$) are analyzed with same procedure and only 93 events are selected whereas the number of signal is 493813. It is negligible.

230 PE/active layer selection

To estimate the uncertainty of PE/active layer selection, we changed the cut value and checked the difference of number of selected events from one with nominal cut. The result is the difference is less than 0.01% and its uncertainty is negligible.

number of active planes	DATA	MC	DATA - MC
3	87.6	86.9	0.7
4	93.2	91.8	1.4
5	94.7	94.3	0.5
6	95.6	96.2	0.6
7	96.2	96.6	0.4
8	96.7	96.8	0.1
9	98.7	97.9	0.8
10	99.1	99.0	0.1
Systematic error (Maximum)			1.4

Table 5.5: The tracking efficiency of DATA and MC with several sub sample

235 beam timing selection

To estimate the uncertainty from neutrino beam timing, we changed the cut value and checked the difference of number of events from nominal cut. The difference is less than 0.01% and it is negligible.

²³⁹ Chapter 6

Measurement of beamprofile

²⁴² 6.1 Stability of beam profile

Fig.6.1 shows horizontal and vertical beam profile with RUN 32 data. Fitted center with gaussian is 0.1 ± 2.9 cm for horizontal and -10.9 ± 3.2 cm for vertical. Fig.6.4 and 6.5 show the monthly beam center of horizontal and vertical respectively. We succeeded to measure the profile center with about 4.2 cm statistical error for each month. The chi-square is calculated to be almost one to the degree of freedom (0.8 for X and Y center). It is concluded that the beam profile center is stable within statistical error.

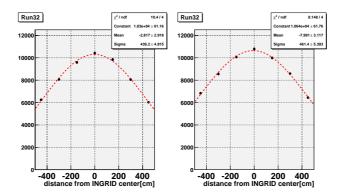


Figure 6.1: Horizontal profile(left) and vertical profile(right)

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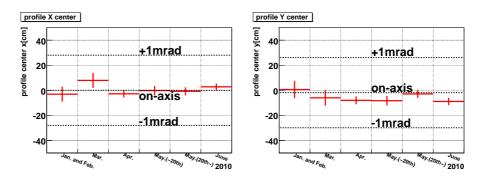


Figure 6.2: Horizontal profile center. Figure 6

Figure 6.3: Vertical profile center

²⁵⁰ 6.2 The systematic error of beam center

We estimated the systematic error with toy profile MC in which the number of events at each module is varied with 3.7% from original profile made by RUN 29 34 all data. 100'000 profiles are generated and the RMS of fitted center is applied as systematic error. The result shows 9.2 cm (0.33 mrad) for horizontal center and 10.4 cm (0.37 mrad) for vertical center.

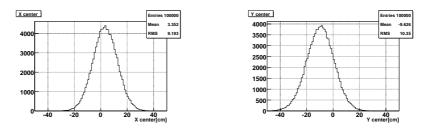


Figure 6.4: Fitted Horizontal center Figure 6.5: Fitted Vertical center with with 100'000 profiles 100'000 profiles

²⁵⁶ Chapter 7 ²⁵⁷ Conclusion

In this note we have presented the measurement of the neutrino event rate and profile center in INGRID durint 2010a. We selected the neutrino interactions to reconstruct the long track started within fiducial volume. The results have been compared to MC and found good agreement with DATA. Finally DATA/MC of the event rate and beam profile centers have been evaluated with an associated systematic error:

$R_{DATA/MC}$	$= 1.072 \pm 0.001$ (stat.) ± 0.040 (syst.)	
Xcenter	$= +0.2 \pm 1.4(\text{stat}) \pm 9.2(\text{syst.})$	cm
Ycenter	$= -6.6 \pm 1.5 (\text{stat.}) \pm 10.4 (\text{syst})$	cm

²⁶⁴ Appendix A

²⁶⁵ Correction factors for ²⁶⁶ neutrino event rate

267 Iron mass

In INGRID most of the neutrino interactions occur in the 9 iron targets of each 268 module. During their fabrication, there was a tolerance on the thickness of those 269 iron planes. This results in iron planes having slightly different volumes, and 270 as a consequence different masses. The maximal variation from design mass is 271 2.15 % from the given tolerance on thickness. The mass of each iron plane was 272 measured at the end of the fabrication process, so we can deduce correction 273 factors for the expected number of events for each module, by using the fact 274 that 95.2 % of interactions in one module occur in the iron. 275

276 Accidental MPPC noise

Another correction on the number of observed events comes from noise hits
in the detector. Those noise hits reduce the number of reconstructed events
compared to the case when there is no noise. To correct this effect, we use the
following procedure:

- 281 (1) Measure noise in data
- 282 (2) Create a noise simulation to reproduce those measurements
- (3) Use Monte Carlo simulation to compare the number of reconstructed events
 with and without adding noise
- ²⁸⁵ (4) Deduce from the simulation correction factors and systematic errors
- Noise is measured in beam data. We measure the rate of noise hits, which are defined as hits occuring in the detector when no particles are actually going through the detector. To find such hits, we look at cycles where beam spills are not coming (INGRID records data on 23 integration cycles, but beam spills

only arrive during 6 of them), and perform regular event selection to make sure
there is no cosmic particle in the detector. We then measure a noise rate for
each channel of the detector, as well as light yield and timing distribution.

Noise is then simulated with a given probability for each channel. Timing for
the noise hits is simulated using the distribution measured in data. Light yield
is then simulated using a measured light yield distribution for the corresponding
timing.

Monte Carlo simulation is then used to measure the variation of number of reconstructed events due to noise. We first reconstruct events on Monte Carlo files which do not include noise hits, then add noise hits to those files and perform reconstruction again. We then compare the number of reconstructed events in each case. The simulation is using jnubeam 10c.

From this simulation we have for each module a noise rate and the variation of number of reconstructed events due to noise. There is a linear relation between them as can be seen on Fig.A.1. We will use this linear relation to make corrections on the number of observed events. This relation is:

Variation of number of events [%] = -0.9585 * < noise rate >

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Those corrections are made for each module, and each subset of events we are considering. In each case we measure the noise rate, and then from the linear relation deduce the variation of number of reconstructed events which should be used as a correction factor.

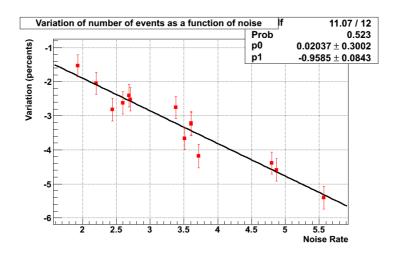


Figure A.1: Variation of number of reconstructed events as a function of noise rate

311 Beam related background

³¹² We estimated the contamination fraction of beam related background with back-

ground MC in which neutrino and the interaction is generated in upstream dirt ($10 \ge 10 \ge 5 = m^3$).

Almost all contaminations come from short track induced neutron ($\sim 50\%$) or gamma ($\sim 40\%$) and dirt muon ($\sim 10\%$) which is not detected accidentally due to scintillator inefficiency.

In background MC, number of generated interactions is normalized so that number of rejected events at upstream VETO selection, which consists dirt muon mainly, is equal to DATA and MC. The difference from POT expectation is 35% and it is considered as systematic error. Finally contamination fraction setimated to be 0.4% and it is applied as one of the correction factor.

323 Summary of the correction factor

Run by run and module by module correction factors are summarized int Table
 A.1

	module	20.20	31	32	33	33	34
number of events	module 0	29,30			4119		
number of events	0	$1054 \\ 1526$	1548	5956	4119 5432	5425 7122	$6962 \\ 9520$
	$\frac{1}{2}$		2033	7827		7122 8555	
	$\frac{2}{3}$	1875	2476	$9360 \\ 10133$	6492	8555	11622
		1882	2570		6795 6626	9078	12191
	4	1831	2459	9627	6636 5491	8683	11651
	5	1524	2176	7876	5421	7217	9588 7285
	6	1058	1585	5837	4172	5421	7285
	7	1229	1717	6636	$4509 \\ 5819$	5826 7620	8100
	8	1588	2187	8351	6632	7620	10270
	9	1884	2562	9770		8766	11946
	10	1949	2681	10305	6987 6712	9373	12473
	11	1908	2520	9771	6713	8897	11871
	12	1561	2133	8146	5512	7193	9822 7724
fff	13	1218	1659	6263	4327	5815	7734
correction factor	0	-3.3	-3.3	-4.3	-4.0	-3.9	-3.9
	1	-2.6	-2.6	-2.6	-2.4	-2.4	-2.4
	$\frac{2}{3}$	-2.0	-2.0	-2.0	-1.7	-1.7	-1.7
		-2.3	-2.3	-2.3	-2.0	-1.9	-1.9
	4	-1.8	-1.8	-1.8	-1.6	-1.5	-1.5
	5	-1.9	-1.9	-1.9	-1.6	-1.5	-1.4
	6	-2.3	-2.3	-2.8	-2.5	-2.4	-2.3
	7	-2.7	-2.7	-2.5	-3.5	-3.3	-3.1
	8	-2.2	-2.2	-2.0	-3.0	-2.8	-2.6
	9	-2.1	-2.1	-2.7	-4.1	-3.8	-3.6
	10	-4.2	-4.2	-4.1	-5.4	-5.2	-5.0
	11	-1.9	-1.9	-1.8	-2.9	-2.7	-2.6
	12	-4.9	-4.9	-4.7	-6.2	-6.0	-5.9
Number of our	13	-2.5	-2.5	-2.5	-3.4	-3.2	-3.0
Number of cor.	0	1090	1601	6224	4292	5647	7242
	1	1567	2087	8037	5565 6606	7298	9756
	2	1913	2526	9551	6606	8703	11822
	3	1927	2632	10374	6934 6741	9255	12422
	4	1865	2504	9804	6741	8817	11825
	5	1553	2218	8028	5509	7328	9697 7459
	67	1083	1622	6002	4278	5552 6024	7453
	7	1263	1765	6803	4674	6024 7828	8360 10545
	8	1624 1025	2237	8520	6001 6012	7838	10545
	9 10	1925	2617	10040	6912 7295	9110	12391
	10	2035	2800	10742	7385	9883	13130
	11	1945	2569	9951	6914	9143	12183
	12	1641	2242	8550	5878	7653	10435
	13	1250	1702	6421	4478	6005	7976

Table A.1: Correction factors