実験ゼミ

Technology for Nuclear and Particle Physics

2.8 The Interaction of Neutrons

- Features of Neutron:
  - No electric charge
  - Strong force
  - Short rage ($\mathbf{10^{-15}}\boldsymbol{cm}$)
  - Difficult to observe

2.8 The Interaction of Neutrons

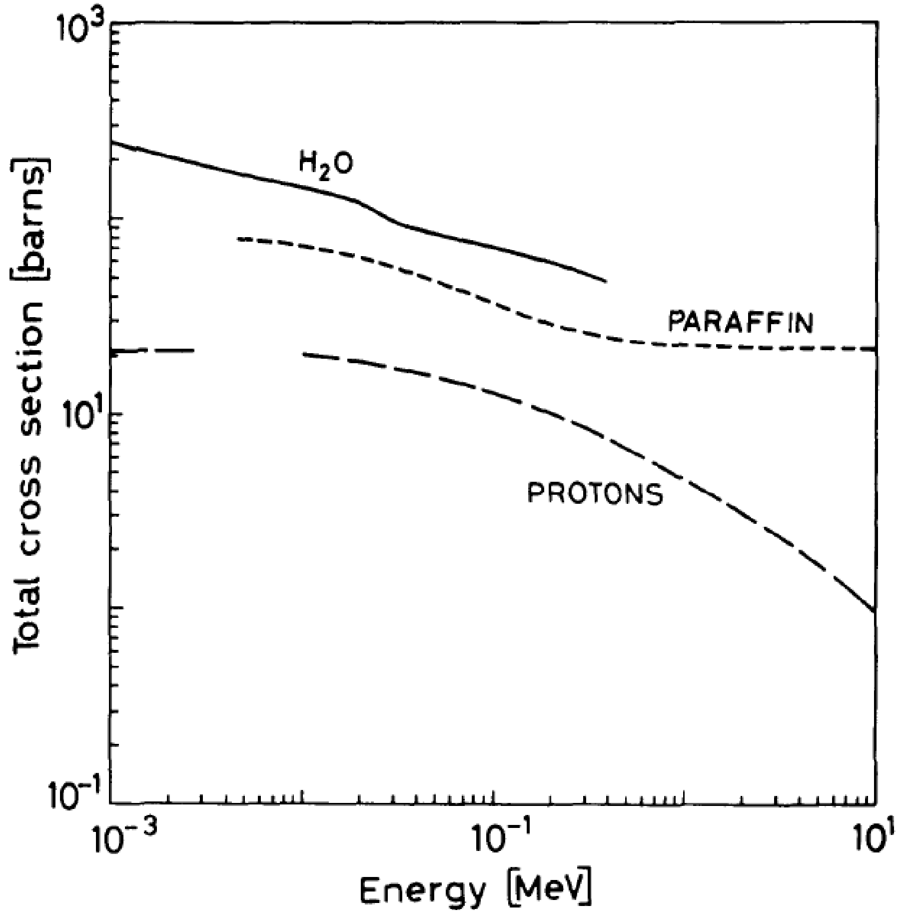
- Nuclear Process
  - Elastic scattering from nuclei (MeV): (n, n)   Principal
  - Inelastic scattering (>1MeV): (n, n'), (n, 2n')
  - Radiative neutron capture (Low energies): n+(Z, A)$\rightarrow\gamma$+(Z, A+1)
  - Other nuclear reactions (eV ~ keV): (n, p),(n, d),(n, α),(n, t)
  - Fission (thermal energy)
  - High energy hadron shower production (>100MeV)

2.8 The Interaction of Neutrons

- Classification
  - High energy neutron: above 100MeV
  - Fast neutron: a few hundred keV to a few ten's MeV
  - Epithermal neutron: between 0.1eV and 100keV
  - Thermal or slow neutron: room temperature(~1/40eV)
  - Cold or ultra-cold neutron: milli- or micro-eV

2.8 The Interaction of Neutrons

- This figure gives an example of the total reaction cross-section for neutrons on a few materials versus neutron energy.
- Here the energy dependence is quite smooth.



2.8 The Interaction of Neutrons

- The mean free path length is:
- $\frac{1}{\lambda} = N\sigma_{tot} = \frac{N_a \rho}{A}\sigma_{tot}$ (1)
- Where $\sigma_{tot} = \sigma_{elastic} + \sigma_{inelastic} + \sigma_{capture} + \cdots$
- A beam of neutrons passing through matter will be:
- $N = N_0 e^{-\frac{x}{\lambda}}$ (2)
- Equation (2) is useful only for a collimated beam of neutrons.

2.8.1 Slowing Down of Neutrons: Moderation

- A fast neutron will scatter on the nuclei both elastically and inelastically after entering into matter and lose energy until it comes into thermal equilibrium with the surrounding atoms.
- It will then diffuse through matter until being captured by a nucleus or enters into other type of nuclear reaction like fission.
- It may also undergo a nuclear reaction or be captured before attaining thermal energy, especially when resonances are present.
- However, without such reaction, neutrons down to thermal velocities are likely to survive duto the v^{-1} dependence of the cross-section.

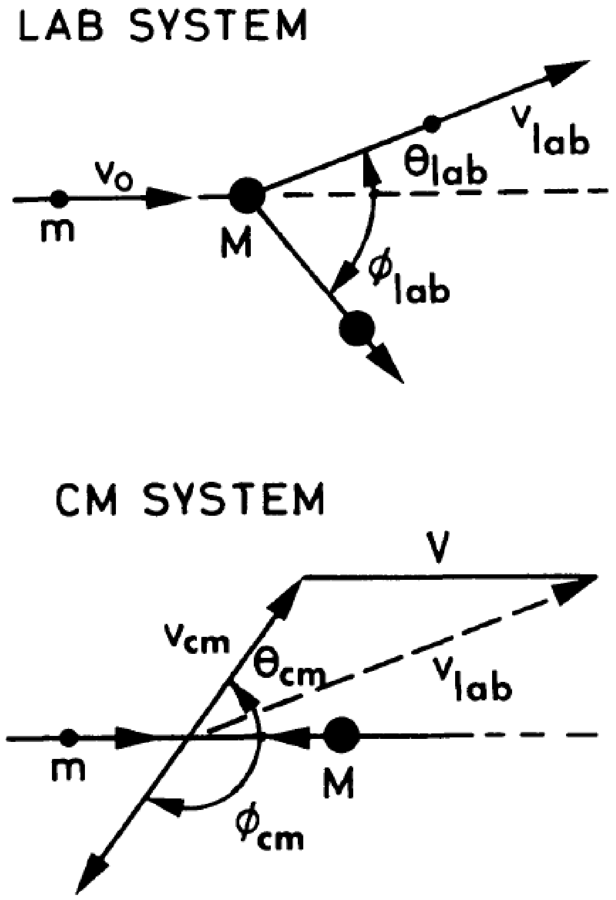
2.8.1 Slowing Down of Neutrons: Moderation

- At energies of several MeV, elastic scattering is the principal mechanism of energy loss for fast neutrons and they can be considered to be nonrelativistic.
- In the lab frame of reference, a neutron with velocity v_0 scatter with a nucleus at rest with a mass M.
- In units of neutron mass, $m_n = 1$ and the mass of the nucleus is the atomic mass number A.

2.8.1 Slowing Down of Neutrons: Moderation

- Transform into center-of-mass system, velocity of the neutron becomes:
- $v_{cm} = \frac{A}{A+1} v_0$ (3)

- The nucleus takes on a velocity:
- $V = \frac{1}{A+1} v_0$ (4)

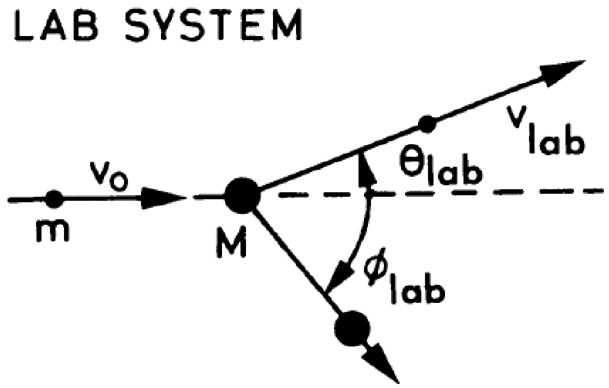


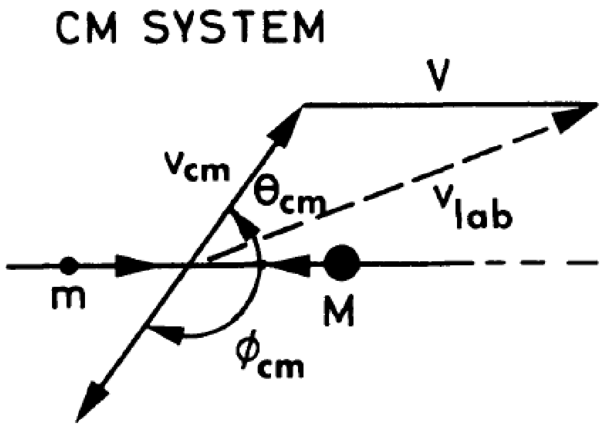
2.8.1 Slowing Down of Neutrons: Moderation

- The speed of the neutron stays the same but the direction changes in cm system.
- The corresponding velocity of the neutron in lab system is:

- $(v_{lab})^2 = (v_{cm})^2 + V^2 - 2v_{cm}V\cos(\pi - \theta_{cm})$ (5)

- where θ_{cm} is the cm scattering angle.

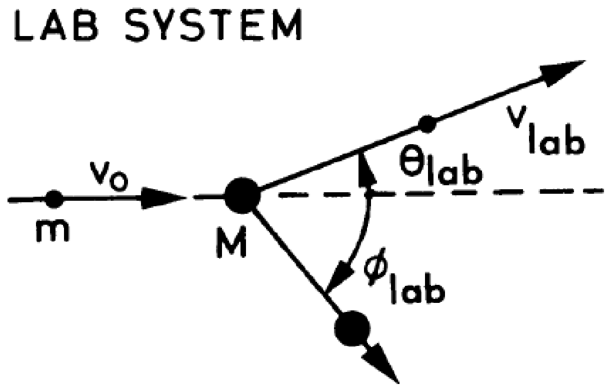


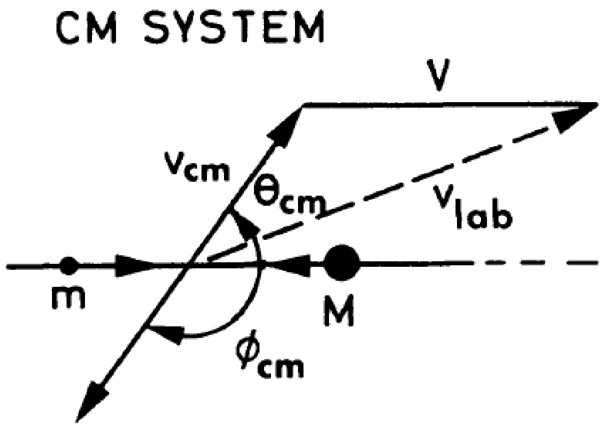


2.8.1 Slowing Down of Neutrons: Moderation

- Substitute (3) and (4) into (5):
- $(v_{lab})^2 = \left(\frac{A}{A+1}\right)^2 v_0^2 + \left(\frac{1}{A+1}\right)^2 v_0^2 - 2\frac{A}{(A+1)^2} v_0^2 \cos(\pi - \theta_{cm})$ (6)

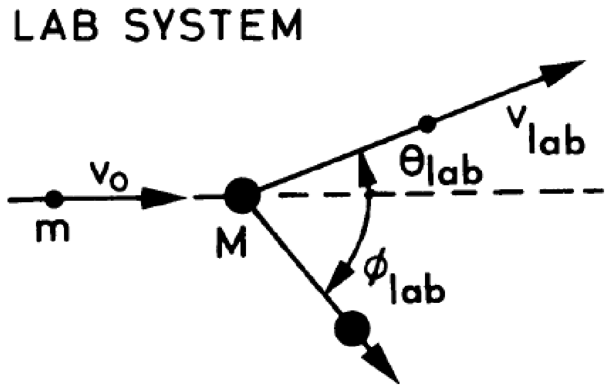
- Since $E = 1/2mv^2$, we have:
- $\frac{E}{E_0} = \left(\frac{v_{lab}}{v_0}\right)^2 = \frac{A^2+1+2A\cos\theta_{cm}}{(A+1)^2}$ (7)

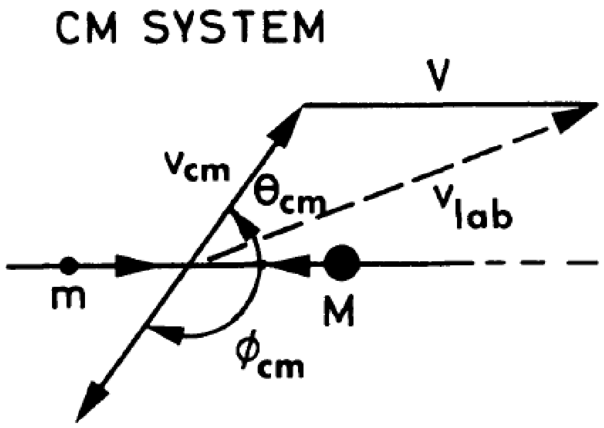




2.8.1 Slowing Down of Neutrons: Moderation

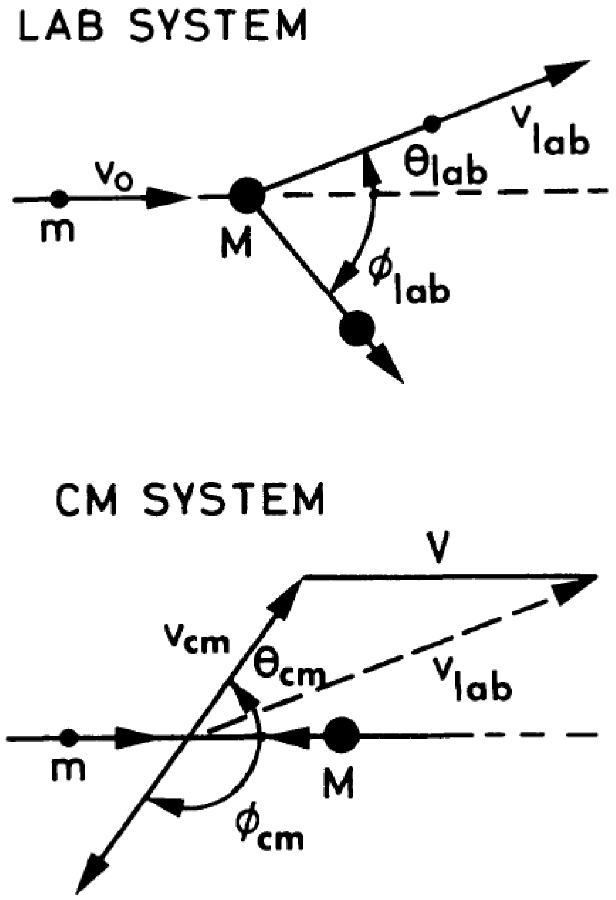
- The lab scattering angle θ_{lab}:
- $(v_{cm})^2 = (v_{lab})^2 + V^2 - 2v_{lab}V\cos\theta_{lab}$ (8)
- Which using (6) yields:
- $\cos\theta_{lab} = \frac{A\cos\theta_{cm}+1}{\sqrt{A^2+1+2A\cos\theta_{cm}}}$ (9)





2.8.1 Slowing Down of Neutrons: Moderation

- The scattering parameters for the recoil nucleus:
- $E_A = E_0 \frac{4A}{(A+1)^2} \cos^2 \phi_{lab} = E_0 \frac{2A}{(A+1)^2} (1 + \cos \phi_{cm})$ (10)
- $\cos \phi_{lab} = \sqrt{\frac{1+\cos \phi_{cm}}{2}}$ or $\phi_{lab} = \frac{1}{2} \phi_{cm}$ (11)



2.8.1 Slowing Down of Neutrons: Moderation

- The energy of the scattered neutron is limited in:
- $\left(\frac{A-1}{A+1}\right)^2 E_0 < E < E_0$ (12)
- The limits correspond to scattering at $\theta_{cm} = \pm 1$
- In particular case of scattering on protons, $A = 1$:
- $0 < E < E_0$
- The lighter the nucleus is the more recoil energy it absorbs.
- That is why hydrogenous materials such as water or paraffin are used in neutron moderation and shielding.

2.8.1 Slowing Down of Neutrons: Moderation

- At not too high energies ($\leq 15MeV$), neutron scattering is restricted to s-wave scattering, which is isotropic, so the probability of scattering into a solid angel $d\Omega$ is:
- $dw = \frac{d\Omega}{4\pi} = 2\pi \sin\theta_{cm} \frac{d\theta_{cm}}{4\pi} = \frac{1}{2} \sin\theta_{cm}\, d\theta_{cm}$ (13)

- From (7), however:
- $\frac{dE}{E_0} = 2\frac{A}{(A+1)^2} \sin\theta_{cm}\, d\theta_{cm}$ (14)

2.8.1 Slowing Down of Neutrons: Moderation

- After substitution, it yields:
- $\frac{dw_1}{dE} = \frac{(A+1)^2}{4A}\frac{1}{E_0} = \frac{1}{E_0(1-\alpha)}$ (15)
- Where
- $\alpha = [(A-1)/(A+1)]^2$
- The energy distribution of an originally monoenergetic neutrino is constant over the energy range given by (12).

2.8.1 Slowing Down of Neutrons: Moderation

- The distribution of two scattering
- $\frac{dw_2}{dE} = \begin{cases} \int_E^{E_0} d\varepsilon \frac{dw_1}{d\varepsilon} \frac{1}{\varepsilon(1-\alpha)} = \frac{1}{E_0(1-\alpha)^2} \ln \frac{E_0}{E} & \alpha E_0 < E < E_0 \\ \int_{\alpha E_0}^{E/\alpha} d\varepsilon \frac{dw_1}{d\varepsilon} \frac{1}{\varepsilon(1-\alpha)} = -\frac{1}{E_0(1-\alpha)^2} \left[\ln \frac{E_0}{E} + 2 \ln \alpha \right] & \alpha^2 E_0 < E < \alpha E_0 \end{cases}$ (16)

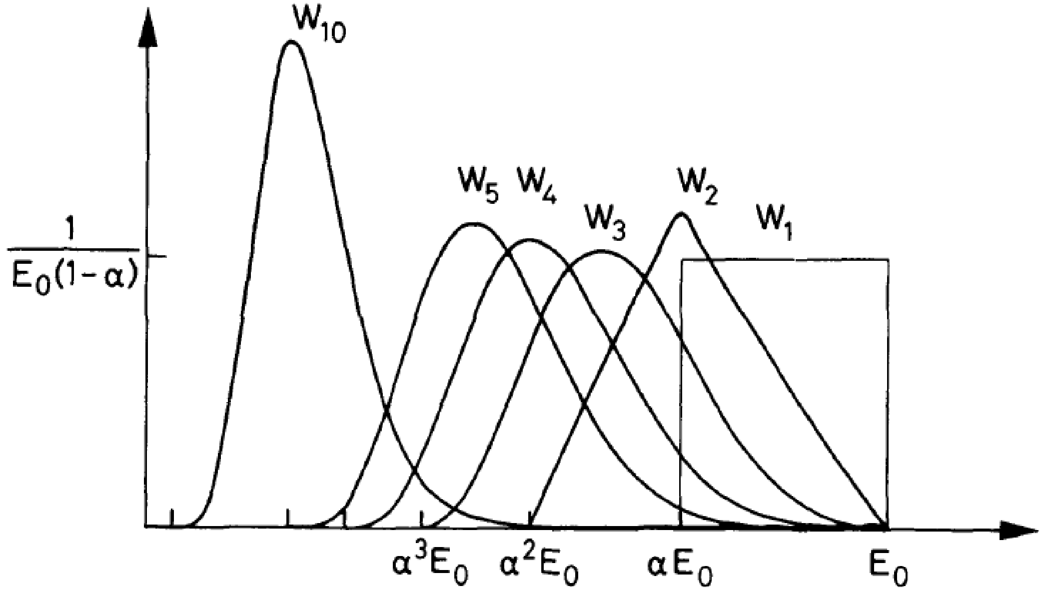
2.8.1 Slowing Down of Neutrons: Moderation

- Three scattering
- $\frac{dw_3}{dE} =$

$$\begin{cases} \frac{1}{2E_0(1-\alpha)^3}\left(\ln\frac{E_0}{E}\right)^2 & \alpha E_0 < E < E_0 \\ -\frac{1}{2E_0(1-\alpha)^3}\left[2\left(\ln\frac{E_0}{E}\right)^2 + 6\ln\alpha\ln\frac{E_0}{E} + 3(\ln\alpha)^3\right] & \alpha^2 E_0 < E < \alpha E_0 \\ \frac{1}{2E_0(1-\alpha)^3}\left(\ln\frac{E_0}{E} + 3\ln\alpha\right)^3 & \alpha^3 E_0 < E < \alpha^2 E_0 \end{cases} \tag{17}$$

2.8.1 Slowing Down of Neutrons: Moderation

- For more collisions it can be worked out by continuing it.
- The figure compares the distribution obtained after several collisions
- For the case of n scattering on hydrogen:
- $\frac{dw_n}{dE} = \frac{1}{E_0(n-1)!}\left(\ln\frac{E_0}{E}\right)^{n-1}$ (18)



2.8.1 Slowing Down of Neutrons: Moderation

- How many collisions are needed to reduce the average energy of a neutrino to some given level?
- $u = \ln E_0 - \ln E = \ln \frac{E_0}{E}$ (19)
- Where E_0 is the initial energy and E the final energy. It is also known as lethargy change.

- After a single scattering at angle θ, u is
- $u(\theta) = \ln \frac{(A+1)^2}{A^2+1+2A\cos\theta}$ (20)

2.8.1 Slowing Down of Neutrons: Moderation

- Integrate (20) over all directions and divide by 4π, we can find the average $u(\theta)$ for a single scattering
- $\xi = \langle u(\theta) \rangle = \int u(\theta) \frac{d\Omega}{4\pi} = \frac{1}{2} \int \ln \frac{(A+1)^2}{A^2+1+2A\cos\theta} d(\cos\theta) = 1 + \frac{(A-1)^2}{2A} \ln \frac{A-1}{A+1}$ (21)
- The average lethargy after one scattering is a constant. The average number of collision n required for this would be:
- $n = \frac{u}{\xi} = \frac{1}{\xi} \ln \frac{E_0}{E'}$ (22)

2.8.1 Slowing Down of Neutrons: Moderation

- For moderator of carbon-12, $\xi = \mathbf{0.158}$, so the number of collisions needed for a 1 MeV neutron to slow down to thermal energy (1/40 eV) is $\cong \mathbf{111}$.

- For hydrogen,$\xi = \mathbf{1}$, this number would only be $\cong \mathbf{17.5}$