実験ゼミ

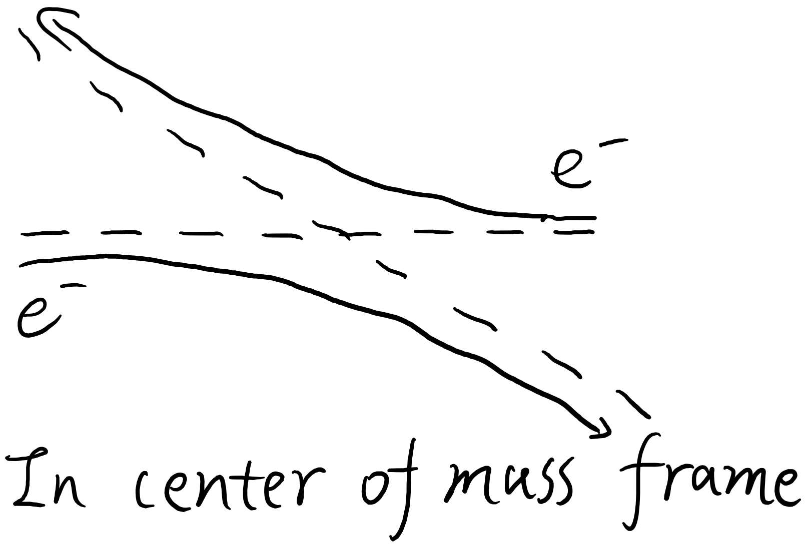
Technology for Nuclear and Particle Physics

2.4 Energy Loss of Electron and Positron

- Composition
  - Collision
  - Bremsstrahlung

- Condition for Bremsstrahlung
  - A few MeV or less: relatively small factor
  - Up to 10's of MeV: comparable to or greater
  - Critical energy: $\left(\frac{dE}{dx}\right)_{rad} = \left(\frac{dE}{dx}\right)_{coll}$

2.4.1 Collision Loss



- Similar to heavy charged particles
- Modified Bethe-Bloch formula
  - Small mass
  - Identical particles

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2} + F(\tau) - \delta - 2\frac{C}{Z}\right]$$

2.4.1 Collision Loss

- $-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2} + F(\tau) - \delta - 2\frac{C}{Z} \right]$
  - For e^-: $F(\tau) = 1 - \beta^2 + \frac{\frac{\tau^2}{8} - (2r+1)\ln 2}{(\tau+1)^2}$
  - For e^+: $F(\tau) = 2\ln 2 - \frac{\beta^2}{12}\left(23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3}\right)$
    - τ: Kinetic energy of particle in units of $m_e c^2$.
    - I: Mean excitation potential.
    - δ: Density effect.
    - C: Shell effect.

2.4.1 Collision Loss

- Comparison between electron & heavy particles
  - Electron (positron):
    - $-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2} + F(\tau) - \delta - 2\frac{C}{Z}\right]$
  - Heavy particles:
    - $-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\frac{2m_e\gamma^2 v^2 W_{max}}{I^2} - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$

2.4.2 Energy Loss by Radiation: Bremsstrahlung

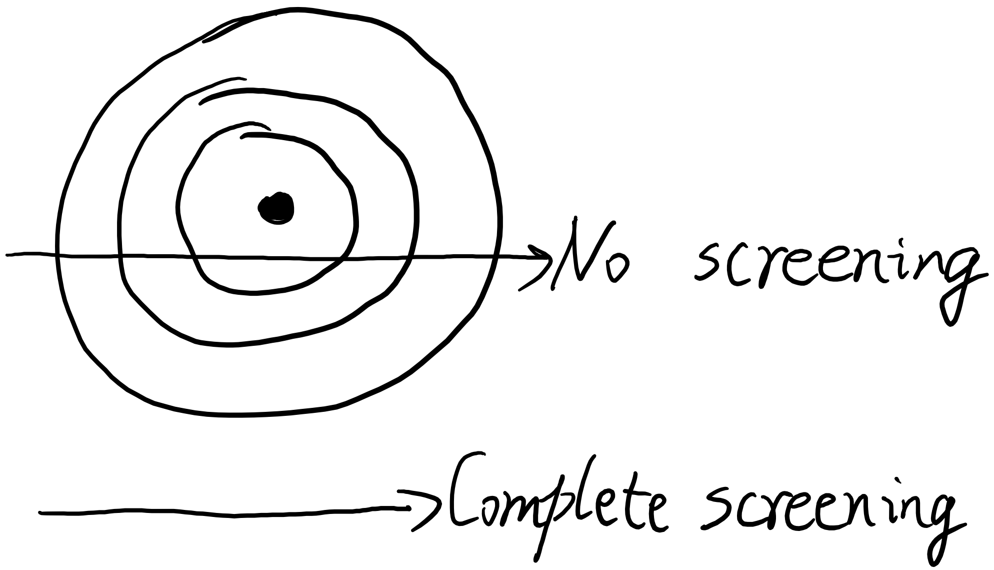
- The Bremsstrahlung is not considered for the heavy particle since it is quite a small factor. The emission probability varies as the inverse square of the particle mass: $\sigma \propto \left(e^2/mc^2\right)^2$.
- For example:
  - The mass of the electron is $m_e \cong 0.51\ \mathrm{MeV}$, the mass of the next lightest particle muon is $m_\mu \cong 106\ \mathrm{MeV}$, the ratio of the probability is $\sigma_e/\sigma_\mu = m_\mu^2/m_e^2 \cong 40{,}000$.
- For proton or alpha particle, it is even smaller.

2.4.2 Energy Loss by Radiation: Bremsstrahlung

- Since bremsstrahlung emission depends on the strength of electric field felt by the electron, the amount of screening from the atomic electrons surrounding the nucleus plays an important role.
- Screening:
  - Although the electric field outside the atom is zero, it is not inside the atom. The atomic electrons between the incident electron and the nucleus screen the electric field from the nucleus, but those outside do not. So the effect of screen differs as the distance between the incident particle and the nucleus changes.

2.4.2 Energy Loss by Radiation: Bremsstrahlung

- Effect of screening: $\xi = \frac{100 m_e c^2 h\upsilon}{E_0 E Z^{1/3}}$
- E_0: initial total energy of electron; E: final energy of electron; $h\upsilon$: energy of photon emitted; Z: atomic number.
- Complete screening: $\xi \approx 0$; no screening: $\xi \gg 1$.



2.4.2 Energy Loss by Radiation: Bremsstrahlung

- Bremsstrahlung cross section for relativistic energies greater than a few MeV:
- $d\sigma = 4Z^2 r_e^2 \alpha \frac{d\nu}{\nu}\left\{\left(1+\varepsilon^2\right)\left[\frac{\phi_1(\xi)}{4} - \frac{1}{3}\ln Z - f(Z)\right] - \frac{2}{3}\varepsilon\left[\frac{\phi_2(\xi)}{4} - \frac{1}{3}\ln Z - f(Z)\right]\right\}$
- ε: E_0/E, α: $1/137$, $f(Z)$: Coulomb correction, $\phi_1(\xi)$, $\phi_2(\xi)$: screening function.
- Note: not valid for low energy.

2.4.2 Energy Loss by Radiation: Bremsstrahlung

- Empirical formulae for $Z \geq 5$:
- $\phi_1(\xi) = 20.863 - 2\ln[1 + (0.55846\xi)^2] - 4[1 - 0.6\exp(-0.9\xi) - 0.4\exp(-1.5\xi)]$
- $\phi_2(\xi) = \phi_1(\xi) - \frac{2}{3}\left(1 + 6.5\xi + 6\xi^2\right)^{-1}$
- Approximation accurate to $\cong 0.5\%$.
- $\phi_1(0) = \phi_2(0) + \frac{2}{3} = 4\ln 183$
- $\phi_1(\infty) = \phi_2(\infty) \rightarrow 19.19 - 4\ln\xi$

2.4.2 Energy Loss by Radiation: Bremsstrahlung

- Small correction to the Born approximation:
- $f(Z) \cong a^2 \left[\left(1 + a^2\right)^{-1} + 0.20206 - 0.0369a^2 + 0.0083a^4 - 0.002a^6 \right]$
- $a = Z/137$
- Taking into account the Coulomb interaction of the emitting electron in the electric field of the nucleus.

2.4.2 Energy Loss by Radiation: Bremsstrahlung

- For $\xi \gg 1$:
- $d\sigma = 4Z^2 r_e^2 \alpha \frac{dv}{v}\left(1 + \varepsilon^2 - \frac{2\varepsilon}{3}\right)\left[\ln\frac{2E_0E}{m_e c^2 h\nu} - \frac{1}{2} - f(Z)\right]$
- For $\xi \approx 1$:
- $d\sigma = 4Z^2 r_e^2 \alpha \frac{dv}{v}\left\{\left(1 + \varepsilon^2 - \frac{2\varepsilon}{3}\right)\left[\ln(183Z^{-1/3}) - f(Z)\right] + \frac{\varepsilon}{9}\right\}$
- The energy loss due to radiation can be calculated:
- $-\left(\frac{dE}{dx}\right)_{rad} = N\int_0^{\nu_0} h\nu \frac{d\sigma}{d\nu}(E_0, \nu)d\nu$
- $N = \rho N_a/A$; $\nu_0 = E_0/h$

2.4.2 Energy Loss by Radiation: Bremsstrahlung

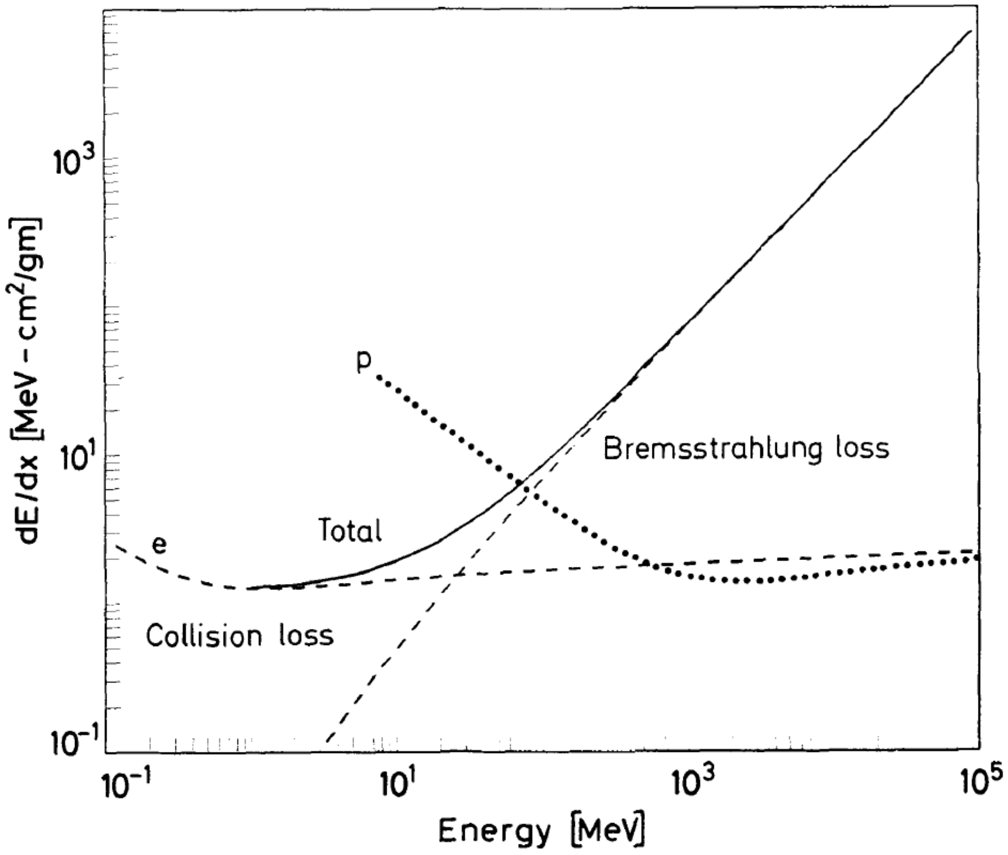
- It can be rewritten as:
- $-\left(\frac{dE}{dx}\right)_{rad} = NE_0\Phi_{rad}$
- $\Phi_{rad} = \frac{1}{E_0}\int h\nu\frac{d\sigma}{d\nu}(E_0,\nu)d\nu$
- $d\sigma/d\nu$ is approximately proportional to ν^{-1} so the integral Φ_{rad} is independent of ν and is a function of the material only.

2.4.2 Energy Loss by Radiation: Bremsstrahlung

- For $m_e c^2 \ll E_0 \ll 137 m_e c^2 Z^{-1/3}$, $\xi \gg 1$:
- $\Phi_{rad} = 4Z^2 r_e^2 \alpha \left(\ln \frac{2E_0}{m_e c^2} - \frac{1}{3} - f(Z) \right)$
- For $E_0 \gg 137 m_e c^2 Z^{-1/3}$, $\xi \approx 1$:
- $\Phi_{rad} = 4Z^2 r_e^2 \alpha \left[\ln(183 Z^{-1/3}) + \frac{1}{18} - f(Z) \right]$
- At intermediate values of ξ, this must be integrated mumerically.

2.4.2 Energy Loss by Radiation: Bremsstrahlung

- The ionization loss varies logarithmically with E and linearly with Z, while the radiation loss increases almost linearly with E and quadratically with Z.
- Where as the ionization loss is quasi-continuous along the path of the electron, almost all the radiation energy can be emitted in one or two photons.



2.4.3 Electron-Electron Bremsstrahlung

- Apart from the nucleus, the field of the atomic electrons also contributes to the bremsstrahlung.
- The formulae are the same, except for that the Z^2 is replace by Z.
- $d\sigma = 4Zr_e^2\alpha\frac{d\nu}{\nu}\left\{\left(1+\varepsilon^2\right)\left[\frac{\phi_1(\xi)}{4}-\frac{1}{3}\ln Z-f(Z)\right]-\frac{2}{3}\varepsilon\left[\frac{\phi_2(\xi)}{4}-\frac{1}{3}\ln Z-f(Z)\right]\right\}$
- The overall modification can be done by replacing Z^2 with $Z(Z+1)$.
- $d\sigma = 4Z(Z+1)r_e^2\alpha\frac{d\nu}{\nu}\left\{\left(1+\varepsilon^2\right)\left[\frac{\phi_1(\xi)}{4}-\frac{1}{3}\ln Z-f(Z)\right]-\frac{2}{3}\varepsilon\left[\frac{\phi_2(\xi)}{4}-\frac{1}{3}\ln Z-f(Z)\right]\right\}$

2.4.4 Critical Energy

- E_c: the energy at which the radiation loss equals the collision loss.
- $\left(\frac{dE}{dx}\right)_{rad} = \left(\frac{dE}{dx}\right)_{coll}$
- $E > E_c$: $\left(\frac{dE}{dx}\right)_{rad} > \left(\frac{dE}{dx}\right)_{coll}$
- $E < E_c$: $\left(\frac{dE}{dx}\right)_{rad} < \left(\frac{dE}{dx}\right)_{coll}$
- Approximation:
- $E_c \cong \frac{800\text{MeV}}{Z+1.2}$

Material	Critical energy [MeV]
Pb	9.51
Al	51.0
Fe	27.4
Cu	24.8
Air (STP)	102
Lucite	100
Polystyrene	109
NaI	17.4
Anthracene	105
H_2O	92

SUMMARY

- Energy loss for electron and positron
  - Collision
    - Bethe-Block formula
      - Identical particle
      - Small mass
  - Bremsstrahlung
    - Mass dependence
    - Screening
    - Nucleus and electron contribution
  - Critical energy