

添字は上付き

↓ $\gamma^5 \gamma^0$ 等のこと

積 *

左\右	I	γ_0	γ_1	γ_2	γ_3	γ_5	σ_{01}	σ_{02}	σ_{03}	σ_{23}	σ_{31}	σ_{12}	γ_{50}	γ_{51}	γ_{52}	γ_{53}
I	I	γ_0	γ_1	γ_2	γ_3	γ_5	σ_{01}	σ_{02}	σ_{03}	σ_{23}	σ_{31}	σ_{12}	γ_{50}	γ_{51}	γ_{52}	γ_{53}
γ_0	γ_0	I	$-i\sigma_{01}$	$-i\sigma_{02}$	$-i\sigma_{03}$	$-\gamma_5$	$i\gamma_1$	$i\gamma_2$	$i\gamma_3$	$-\gamma_{51}$	$-\gamma_{52}$	$-\gamma_{53}$	$-\gamma_5$	$-\sigma_{23}$	$-\sigma_{31}$	$-\sigma_{12}$
γ_1	γ_1	$i\sigma_{01}$	I	$-i\sigma_{12}$	$i\sigma_{31}$	$-\gamma_5$	$i\gamma_0$	γ_{53}	$-\gamma_{52}$	$-\gamma_{50}$	$i\gamma_3$	$-i\gamma_2$	σ_{23}	γ_5	σ_{03}	$-\sigma_{02}$
γ_2	γ_2	$i\sigma_{02}$	$i\sigma_{12}$	I	$-i\sigma_{23}$	$-\gamma_5$	$-\gamma_{53}$	$i\gamma_0$	γ_{51}	$-i\gamma_3$	$-\gamma_{50}$	$i\gamma_1$	σ_{31}	$-\sigma_{03}$	γ_5	σ_{01}
γ_3	γ_3	$i\sigma_{03}$	$-i\sigma_{31}$	$i\sigma_{23}$	I	$-\gamma_5$	γ_{52}	$-\gamma_{51}$	$i\gamma_0$	$i\gamma_2$	$-i\gamma_1$	$-\gamma_{50}$	σ_{12}	σ_{02}	$-\sigma_{01}$	γ_5
γ_5	γ_5	γ_{50}	γ_{51}	γ_{52}	γ_{53}	I	$i\sigma_{23}$	$i\sigma_{31}$	$i\sigma_{12}$	$-i\sigma_{01}$	$-i\sigma_{02}$	$-i\sigma_{03}$	γ_0	γ_1	γ_2	γ_3
σ_{01}	σ_{01}	$-i\gamma_1$	$-i\gamma_0$	$-\gamma_{53}$	γ_{52}	$i\sigma_{23}$	I	$-i\sigma_{12}$	$i\sigma_{31}$	$i\gamma_5$	$i\sigma_{03}$	$-i\sigma_{02}$	$-i\gamma_{51}$	$-i\gamma_{50}$	$-\gamma_3$	γ_2
σ_{02}	σ_{02}	$-i\gamma_2$	γ_{53}	$-i\gamma_0$	$-\gamma_{51}$	$i\sigma_{31}$	$i\sigma_{12}$	I	$-i\sigma_{23}$	$-i\sigma_{03}$	$i\gamma_5$	$i\sigma_{01}$	$-i\gamma_{52}$	γ_3	$-i\gamma_{50}$	$-\gamma_1$
σ_{03}	σ_{03}	$-i\gamma_3$	$-\gamma_{52}$	γ_{51}	$-i\gamma_0$	$i\sigma_{12}$	$-i\sigma_{31}$	$i\sigma_{23}$	I	$i\sigma_{02}$	$-i\sigma_{01}$	$i\gamma_5$	$-i\gamma_{53}$	$-\gamma_2$	γ_1	$-i\gamma_{50}$
σ_{23}	σ_{23}	$-\gamma_{51}$	$-\gamma_{50}$	$i\gamma_3$	$-i\gamma_2$	$-i\sigma_{01}$	$i\gamma_5$	$i\sigma_{03}$	$-i\sigma_{02}$	I	$i\sigma_{12}$	$-i\sigma_{31}$	$-\gamma_1$	$-\gamma_0$	$i\gamma_{53}$	$-i\gamma_{52}$
σ_{31}	σ_{31}	$-\gamma_{52}$	$-i\gamma_3$	$-\gamma_{50}$	$i\gamma_1$	$-i\sigma_{02}$	$-i\sigma_{03}$	$i\gamma_5$	$i\sigma_{01}$	$-i\sigma_{12}$	I	$i\sigma_{23}$	$-\gamma_2$	$-i\gamma_{53}$	$-\gamma_0$	$i\gamma_{51}$
σ_{12}	σ_{12}	$-\gamma_{53}$	$i\gamma_2$	$-i\gamma_1$	$-\gamma_{50}$	$-i\sigma_{03}$	$i\sigma_{02}$	$-i\sigma_{01}$	$i\gamma_5$	$i\sigma_{31}$	$-i\sigma_{23}$	I	$-\gamma_3$	$i\gamma_{52}$	$-i\gamma_{51}$	$-\gamma_0$
γ_{50}	γ_{50}	γ_5	σ_{23}	σ_{31}	σ_{12}	$-\gamma_0$	$i\gamma_{51}$	$i\gamma_{52}$	$i\gamma_{53}$	$-\gamma_1$	$-\gamma_2$	$-\gamma_3$	I	$i\sigma_{01}$	$i\sigma_{02}$	$i\sigma_{03}$
γ_{51}	γ_{51}	$-\sigma_{23}$	$-\gamma_5$	$-\sigma_{03}$	σ_{02}	$-\gamma_1$	$i\gamma_{50}$	γ_3	$-\gamma_2$	$-\gamma_0$	$i\gamma_{53}$	$-i\gamma_{52}$	$-i\sigma_{01}$	I	$i\sigma_{12}$	$-i\sigma_{31}$
γ_{52}	γ_{52}	$-\sigma_{31}$	σ_{03}	$-\gamma_5$	$-\sigma_{01}$	$-\gamma_2$	$-\gamma_3$	$i\gamma_{50}$	γ_1	$-i\gamma_{53}$	$-\gamma_0$	$i\gamma_{51}$	$-i\sigma_{02}$	$-i\sigma_{12}$	I	$i\sigma_{23}$
γ_{53}	γ_{53}	$-\sigma_{12}$	$-\sigma_{02}$	σ_{01}	$-\gamma_5$	$-\gamma_3$	γ_2	$-\gamma_1$	$i\gamma_{50}$	$i\gamma_{52}$	$-i\gamma_{51}$	$-\gamma_0$	$-i\sigma_{03}$	$i\sigma_{31}$	$-i\sigma_{23}$	I

	I	γ_0	γ_1	γ_2	γ_3	γ_5	σ_{01}	σ_{02}	σ_{03}	σ_{23}	σ_{31}	σ_{12}	γ_{50}	γ_{51}	γ_{52}	γ_{53}
+	I	γ_0	$-\gamma_1$	$-\gamma_2$	$-\gamma_3$	γ_5	$-\sigma_{01}$	$-\sigma_{02}$	$-\sigma_{03}$	σ_{23}	σ_{31}	σ_{12}	$-\gamma_{50}$	γ_{51}	γ_{52}	γ_{53}
$\wedge -1$	I	γ_0	$-\gamma_1$	$-\gamma_2$	$-\gamma_3$	γ_5	$-\sigma_{01}$	$-\sigma_{02}$	$-\sigma_{03}$	σ_{23}	σ_{31}	σ_{12}	$-\gamma_{50}$	γ_{51}	γ_{52}	γ_{53}
det	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Tr	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

ディラック・1.10の表現

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \beta = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

$$\gamma^0 \equiv \beta, \gamma^i \equiv \beta \alpha_i \quad (i=1,2,3)$$

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3, \sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

1.10の行列

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

積

左\右	I	σ_1	σ_2	σ_3
I	I	σ_1	σ_2	σ_3
σ_1	σ_1	I	$i\sigma_3$	$-i\sigma_2$
σ_2	σ_2	$-i\sigma_3$	I	$i\sigma_1$
σ_3	σ_3	$i\sigma_2$	$-i\sigma_1$	I

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\sigma^{01} = \begin{pmatrix} 0 & i\sigma_1 \\ i\sigma_1 & 0 \end{pmatrix} \quad \sigma^{02} = \begin{pmatrix} 0 & i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix} \quad \sigma^{03} = \begin{pmatrix} 0 & i\sigma_3 \\ i\sigma_3 & 0 \end{pmatrix} \quad \sigma^{23} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix} \quad \sigma^{31} = \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix} \quad \sigma^{12} = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

$$\gamma^5\gamma^0 = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix} \quad \gamma^5\gamma^1 = \begin{pmatrix} -\sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix} \quad \gamma^5\gamma^2 = \begin{pmatrix} -\sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix} \quad \gamma^5\gamma^3 = \begin{pmatrix} -\sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} \quad I = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

交換關係 []/2

左\右	I	γ_0	γ_1	γ_2	γ_3	γ_5	σ_{01}	σ_{02}	σ_{03}	σ_{23}	σ_{31}	σ_{12}	γ_{50}	γ_{51}	γ_{52}	γ_{53}
I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
γ_0	0	0	$-i\sigma_{01}$	$-i\sigma_{02}$	$-i\sigma_{03}$	$-\gamma_{50}$	$i\gamma_1$	$i\gamma_2$	$i\gamma_3$	0	0	0	$-\gamma_5$	0	0	0
γ_1	0	$i\sigma_{01}$	0	$-i\sigma_{12}$	$i\sigma_{31}$	$-\gamma_{51}$	$i\gamma_0$	0	0	0	$i\gamma_3$	$-i\gamma_2$	0	γ_5	0	0
γ_2	0	$i\sigma_{02}$	$i\sigma_{12}$	0	$-i\sigma_{23}$	$-\gamma_{52}$	0	$i\gamma_0$	0	$-i\gamma_3$	0	$i\gamma_1$	0	0	γ_5	0
γ_3	0	$i\sigma_{03}$	$-i\sigma_{31}$	$i\sigma_{23}$	0	$-\gamma_{53}$	0	0	$i\gamma_0$	$i\gamma_2$	$-i\gamma_1$	0	0	0	0	γ_5
γ_5	0	γ_{50}	γ_{51}	γ_{52}	γ_{53}	0	0	0	0	0	0	0	γ_0	γ_1	γ_2	γ_3
σ_{01}	0	$-i\gamma_1$	$-i\gamma_0$	0	0	0	0	$-i\sigma_{12}$	$i\sigma_{31}$	0	$i\sigma_{03}$	$-i\sigma_{02}$	$-i\gamma_{51}$	$-i\gamma_{50}$	0	0
σ_{02}	0	$-i\gamma_2$	0	$-i\gamma_0$	0	0	$i\sigma_{12}$	0	$-i\sigma_{23}$	$-i\sigma_{03}$	0	$i\sigma_{01}$	$-i\gamma_{52}$	0	$-i\gamma_{50}$	0
σ_{03}	0	$-i\gamma_3$	0	0	$-i\gamma_0$	0	$-i\sigma_{31}$	$i\sigma_{23}$	0	$i\sigma_{02}$	$-i\sigma_{01}$	0	$-i\gamma_{53}$	0	0	$-i\gamma_{50}$
σ_{23}	0	0	0	$i\gamma_3$	$-i\gamma_2$	0	0	$i\sigma_{03}$	$-i\sigma_{02}$	0	$i\sigma_{12}$	$-i\sigma_{31}$	0	0	$i\gamma_{53}$	$-i\gamma_{52}$
σ_{31}	0	0	$-i\gamma_3$	0	$i\gamma_1$	0	$-i\sigma_{03}$	0	$i\sigma_{01}$	$-i\sigma_{12}$	0	$i\sigma_{23}$	0	$-i\gamma_{53}$	0	$i\gamma_{51}$
σ_{12}	0	0	$i\gamma_2$	$-i\gamma_1$	0	0	$i\sigma_{02}$	$-i\sigma_{01}$	0	$i\sigma_{31}$	$-i\sigma_{23}$	0	0	$i\gamma_{52}$	$-i\gamma_{51}$	0
γ_{50}	0	γ_5	0	0	0	$-\gamma_0$	$i\gamma_{51}$	$i\gamma_{52}$	$i\gamma_{53}$	0	0	0	0	$i\sigma_{01}$	$i\sigma_{02}$	$i\sigma_{03}$
γ_{51}	0	0	$-\gamma_5$	0	0	$-\gamma_1$	$i\gamma_{50}$	0	0	0	$i\gamma_{53}$	$-i\gamma_{52}$	$-i\sigma_{01}$	0	$i\sigma_{12}$	$-i\sigma_{31}$
γ_{52}	0	0	0	$-\gamma_5$	0	$-\gamma_2$	0	$i\gamma_{50}$	0	$-i\gamma_{53}$	0	$i\gamma_{51}$	$-i\sigma_{02}$	$-i\sigma_{12}$	0	$i\sigma_{23}$
γ_{53}	0	0	0	0	$-\gamma_5$	$-\gamma_3$	0	0	$i\gamma_{50}$	$i\gamma_{52}$	$-i\gamma_{51}$	0	$-i\sigma_{03}$	$i\sigma_{31}$	$-i\sigma_{23}$	0

反交換關係 { }/2

左\右	I	γ_0	γ_1	γ_2	γ_3	γ_5	σ_{01}	σ_{02}	σ_{03}	σ_{23}	σ_{31}	σ_{12}	γ_{50}	γ_{51}	γ_{52}	γ_{53}
I	I	γ_0	γ_1	γ_2	γ_3	γ_5	σ_{01}	σ_{02}	σ_{03}	σ_{23}	σ_{31}	σ_{12}	γ_{50}	γ_{51}	γ_{52}	γ_{53}
γ_0	γ_0	I	0	0	0	0	0	0	0	$-\gamma_{51}$	$-\gamma_{52}$	$-\gamma_{53}$	0	$-\sigma_{23}$	$-\sigma_{31}$	$-\sigma_{12}$
γ_1	γ_1	0	$-I$	0	0	0	0	γ_{53}	$-\gamma_{52}$	$-\gamma_{50}$	0	0	σ_{23}	0	σ_{03}	$-\sigma_{02}$
γ_2	γ_2	0	0	$-I$	0	0	$-\gamma_{53}$	0	γ_{51}	0	$-\gamma_{50}$	0	σ_{31}	$-\sigma_{03}$	0	σ_{01}
γ_3	γ_3	0	0	0	$-I$	0	γ_{52}	$-\gamma_{51}$	0	0	0	$-\gamma_{50}$	σ_{12}	σ_{02}	$-\sigma_{01}$	0
γ_5	γ_5	0	0	0	0	I	$i\sigma_{23}$	$i\sigma_{31}$	$i\sigma_{12}$	$-i\sigma_{01}$	$-i\sigma_{02}$	$-i\sigma_{03}$	0	0	0	0
σ_{01}	σ_{01}	0	0	$-\gamma_{53}$	γ_{52}	$i\sigma_{23}$	$-I$	0	0	$i\gamma_5$	0	0	0	0	$-\gamma_3$	γ_2
σ_{02}	σ_{02}	0	γ_{53}	0	$-\gamma_{51}$	$i\sigma_{31}$	0	$-I$	0	0	$i\gamma_5$	0	0	γ_3	0	$-\gamma_1$
σ_{03}	σ_{03}	0	$-\gamma_{52}$	γ_{51}	0	$i\sigma_{12}$	0	0	$-I$	0	0	$i\gamma_5$	0	$-\gamma_2$	γ_1	0
σ_{23}	σ_{23}	$-\gamma_{51}$	$-\gamma_{50}$	0	0	$-i\sigma_{01}$	$i\gamma_5$	0	0	I	0	0	$-\gamma_1$	$-\gamma_0$	0	0
σ_{31}	σ_{31}	$-\gamma_{52}$	0	$-\gamma_{50}$	0	$-i\sigma_{02}$	0	$i\gamma_5$	0	0	I	0	$-\gamma_2$	0	$-\gamma_0$	0
σ_{12}	σ_{12}	$-\gamma_{53}$	0	0	$-\gamma_{50}$	$-i\sigma_{03}$	0	0	$i\gamma_5$	0	0	I	$-\gamma_3$	0	0	$-\gamma_0$
γ_{50}	γ_{50}	0	σ_{23}	σ_{31}	σ_{12}	0	0	0	0	$-\gamma_1$	$-\gamma_2$	$-\gamma_3$	$-I$	0	0	0
γ_{51}	γ_{51}	$-\sigma_{23}$	0	$-\sigma_{03}$	σ_{02}	0	0	γ_3	$-\gamma_2$	$-\gamma_0$	0	0	0	I	0	0
γ_{52}	γ_{52}	$-\sigma_{31}$	σ_{03}	0	$-\sigma_{01}$	0	$-\gamma_3$	0	γ_1	0	$-\gamma_0$	0	0	0	I	0
γ_{53}	γ_{53}	$-\sigma_{12}$	$-\sigma_{02}$	σ_{01}	0	0	γ_2	$-\gamma_1$	0	0	0	$-\gamma_0$	0	0	0	I

	$\pm i$	β	α_1	α_2	α_3	
I	=	1				
γ^0	=		β			
γ^1	=		β	α_1		
γ^2	=		β		α_2	
γ^3	=		β		α_3	
γ^5	=	$-i$		α_1	α_2	α_3

	$\pm i$	β	α_1	α_2	α_3
σ^{01}	=	i		α_1	
σ^{02}	=	i			α_2
σ^{03}	=	i			α_3
σ^{23}	=	$-i$		α_2	α_3
σ^{31}	=	i		α_1	α_3
σ^{12}	=	$-i$		α_1	α_2

	$\pm i$	β	α_1	α_2	α_3	
$\gamma^5 \gamma^0$	=	i	β	α_1	α_2	α_3
$\gamma^5 \gamma^1$	=	$-i$	β		α_2	α_3
$\gamma^5 \gamma^2$	=	$-i$	β	α_1		α_3
$\gamma^5 \gamma^3$	=	i	β	α_1	α_2	

β, α_i は相互反交換、 $\beta^2 = \alpha_1^2 = \alpha_2^2 = \alpha_3^2 = 1$