・伝播関数のフーリエ変換

$$\underbrace{\psi(x_1)\bar{\psi}(x_2)} = iS_F(x_1-x_2) = \frac{1}{(2\pi)^4}\int d^4p\, e^{-ip(x_1-x_2)}\, i\tilde{S}_F(p) \quad , \quad \tilde{S}_F(p) = \frac{1}{\not{p}-m+i\varepsilon} \qquad (4.4節)$$

$$\underbrace{A^\mu(x_1)A^\nu(x_2)} = iD_F^{\mu\nu}(x_1-x_2) = \frac{1}{(2\pi)^4}\int d^4k\, e^{-ik(x_1-x_2)}\, i\tilde{D}_F^{\mu\nu}(k) \quad , \quad \tilde{D}_F^{\mu\nu}(k) = \frac{-g^{\mu\nu}}{k^2+i\varepsilon} \qquad (5.3節)$$

偶関数 $S_F(x)=S_F(-x)$, $D_F^{\mu\nu}(x)=D_F^{\mu\nu}(-x)$, $\tilde{D}_F^{\mu\nu}(k)=\tilde{D}_F(-k)$

・場の演算子のフーリエ展開

$$\psi(x) = \psi^+(x)+\psi^-(x) = \sum_{r,p}\sqrt{\frac{m}{VE_p}}\left[c_r(p)u_r(p)e^{-ipx} + d_r^\dagger(p)v_r(p)e^{ipx}\right]$$

$$\bar{\psi}(x) = \bar{\psi}^+(x)+\bar{\psi}^-(x) = \sum_{r,p}\sqrt{\frac{m}{VE_p}}\left[d_r(p)\bar{v}_r(p)e^{-ipx} + c_r^\dagger(p)\bar{u}_r(p)e^{ipx}\right]$$

$$A^\mu(x) = A^{\mu+}(x)+A^{\mu-}(x) = \sum_{r,k}\sqrt{\frac{1}{2V\omega_k}}\left[\varepsilon_r^\mu(k)a_r(k)e^{-ikx} + \varepsilon_r^\mu(k)a_r^\dagger(k)e^{ikx}\right]$$

c：電子 (4.3節)
d：陽電子
a：光子 (5.1節)

・状態に対する場の演算子の作用

$$\psi^+(x)\,|e^- p r\rangle = |0\rangle\sqrt{\frac{m}{VE_p}}\,u_r(p)e^{-ipx} \qquad \bar{\psi}^-(x)\,|0\rangle = \sum_{r,p}|e^- p r\rangle\sqrt{\frac{m}{VE_p}}\,\bar{u}_r(p)e^{ipx}$$

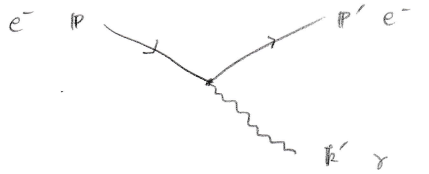
$$\bar{\psi}^+(x)\,|e^+ p r\rangle = |0\rangle\sqrt{\frac{m}{VE_p}}\,\bar{v}_r(p)e^{-ipx} \qquad \psi^-(x)\,|0\rangle = \sum_{r,p}|e^+ p r\rangle\sqrt{\frac{m}{VE_p}}\,v_r(p)e^{ipx}$$

$$A_\mu^+(x)\,|\gamma k r\rangle = |0\rangle\sqrt{\frac{1}{2V\omega_k}}\,\varepsilon_{r\mu}(k)e^{-ikx} \qquad A_\mu^-(x)\,|0\rangle = \sum_{r,k}|\gamma k r\rangle\sqrt{\frac{1}{2V\omega_k}}\,\varepsilon_{r\mu}(k)e^{ikx}$$

$|0\rangle$以外についても同様。スピンと偏極の添字を略す。

7.2.1 1次の項 $S^{(1)}$

例として、右図の散乱過程でS行列を計算する。



(Fig. 7.11)

$$S^{(1)} = ie\int d^4x\, N[\bar{\psi}^-\not{A}^-\psi^+]_x = ie\int d^4x\, \bar{\psi}^-(x)\not{A}^-(x)\psi^+(x)$$

$$|i\rangle = c^\dagger(p)|0\rangle \quad , \quad |f\rangle = c^\dagger(p')a^\dagger(k')|0\rangle$$

$$\langle f|S^{(1)}|i\rangle = \langle 0|\, ie\int d^4x\, a(k')c(p')\,\bar{\psi}^-(x)\not{A}^-(x)\psi^+(x)\,c^\dagger(p)\,|0\rangle$$

$$= ie\int d^4x\left[\sqrt{\frac{m}{VE_{p'}}}\,\bar{u}(p')e^{ip'x}\right]\gamma^\nu\left[\sqrt{\frac{1}{2V\omega_{k'}}}\,\varepsilon_\nu(k')e^{ik'x}\right]\left[\sqrt{\frac{m}{VE_p}}\,u(p)e^{-ipx}\right]$$

$$= \sqrt{\frac{m}{VE_{p'}}}\sqrt{\frac{1}{2V\omega_{k'}}}\sqrt{\frac{m}{VE_p}}\; ie\,\bar{u}(p')\gamma^\nu\varepsilon_\nu(k')u(p)\int d^4x\, e^{i(p'+k'-p)x}$$

$$= \sqrt{\frac{m}{VE_{p'}}}\sqrt{\frac{1}{2V\omega_{k'}}}\sqrt{\frac{m}{VE_p}}\; \underbrace{ie\,\bar{u}(p')\not{\varepsilon}(k')u(p)}_{\mathcal{M}^{(1)}\text{：ファインマン振幅}}\cdot(2\pi)^4\delta^4(k'-p+p')$$

δ関数は頂点におけるエネルギー・運動量保存則を保証する。

始・終状態は実粒子($p_\mu p^\mu = m^2$, $k_\mu k^\mu = 0$)でなければならない。1次の過程は単独では、エネルギー・運動量保存則と両立しないので、実過程ではない。

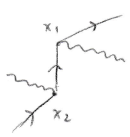
7.2.2 コンプトン散乱ほか

$S_B^{(2)}$ (式(7.5b))から生じる散乱過程には、以下の6種がある。

※ $S_B^{(2)} = -e^2\int d^4x_1\int d^4x_2\, N\left[(\cdots)_{x_1}\gamma^\mu iS_F(x_1-x_2)\gamma^\nu(\cdots)_{x_2}\right]$ の[ ]内を、引数を略して書く。

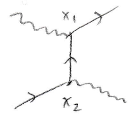
e^-のコンプトン散乱

$S_{B1}^{(2)}$



$\bar\Psi^- A_\mu^- \gamma^\mu iS_F\gamma^\nu A_\nu^+\Psi^+$

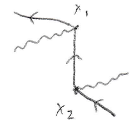
$S_{B2}^{(2)}$



$\bar\Psi^- A_\mu^+ \gamma^\mu iS_F\gamma^\nu A_\nu^-\Psi^+$

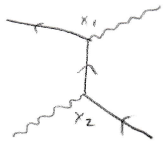
e^+のコンプトン散乱

$S_{B3}^{(2)}$



$\bar\Psi^+ A_\mu^+ \gamma^\mu iS_F\gamma^\nu A_\nu^-\Psi^-$

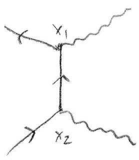
$S_{B4}^{(2)}$



$\bar\Psi^+ A_\mu^- \gamma^\mu iS_F\gamma^\nu A_\nu^+\Psi^-$

e^+e^-対消滅

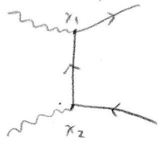
$S_{B5}^{(2)}$



$\bar\Psi^+ A_\mu^- \gamma^\mu iS_F\gamma^\nu A_\nu^-\Psi^+$

e^+e^-対生成

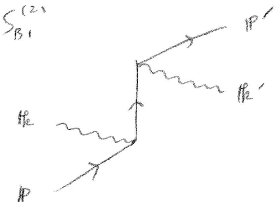
$S_{B6}^{(2)}$



$\bar\Psi^- A_\mu^+ \gamma^\mu iS_F\gamma^\nu A_\nu^+\Psi^-$

• e^-のコンプトン散乱 B1 (Eq. 7.38a)

$S_{B1}^{(2)}$



(Fig. 7.12a)

$$S_{B1}^{(2)} = -e^2\int d^4x_1\int d^4x_2\, N\left[(\bar\Psi^- A_\mu^-)_{x_1}\gamma^\mu iS_F(x_1-x_2)\gamma^\nu(A_\nu^+\Psi^+)_{x_2}\right]$$
$$= -e^2\int d^4x_1\int d^4x_2\, \bar\Psi^-(x_1)A_\mu^-(x_1)\gamma^\mu iS_F(x_1-x_2)\gamma^\nu A_\nu^+(x_2)\Psi^+(x_2)$$

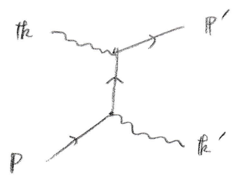
$|i\rangle = c^\dagger(p)a^\dagger(k)|0\rangle$ 、 $|f\rangle = c^\dagger(p')a^\dagger(k')|0\rangle$

$$\langle f|S_{B1}^{(2)}|i\rangle = -e^2\int d^4x_1\int d^4x_2 \left[\sqrt{\frac{m}{VE_{p'}}}\,\bar u(p')e^{ip'x_1}\right]\left[\sqrt{\frac{1}{2V\omega_{k'}}}\,\not\varepsilon(k')e^{ik'x_1}\right]\left[\frac{1}{(2\pi)^4}\int d^4q\, e^{-iq(x_1-x_2)}\, i\tilde S_F(q)\right]$$
$$\left[\sqrt{\frac{1}{2V\omega_k}}\,\not\varepsilon(k)e^{-ikx_2}\right]\left[\sqrt{\frac{m}{VE_p}}\,u(p)e^{-ipx_2}\right]$$
$$= \sqrt{\frac{m}{VE_{p'}}}\sqrt{\frac{1}{2V\omega_{k'}}}\sqrt{\frac{1}{2V\omega_k}}\sqrt{\frac{m}{VE_p}}\ \underbrace{(-e^2)\,\bar u(p')\not\varepsilon(k')\,i\tilde S_F(p+k)\,\not\varepsilon(k)u(p)}_{\mathcal{M}_{B1}^{(2)}}\cdot(2\pi)^4\delta^4(p'+k'-p-k)$$

$\uparrow$
$q = p'+k' = p+k$

• e^-のコンプトン散乱 B2 (Eq. 7.38b)

$S_{B2}^{(2)}$



(Fig. 7.12b)

$$S_{B2}^{(2)} = -e^2\int d^4x_1\int d^4x_2\, N\left[(\bar\Psi^- A_\mu^+)_{x_1}\gamma^\mu iS_F(x_1-x_2)\gamma^\nu(A_\nu^-\Psi^+)_{x_2}\right]$$
$$= -e^2\int d^4x_1\int d^4x_2\, \bar\Psi^-(x_1)A_\nu^-(x_2)\gamma^\mu iS_F(x_1-x_2)\gamma^\nu A_\mu^+(x_1)\Psi^+(x_2)$$

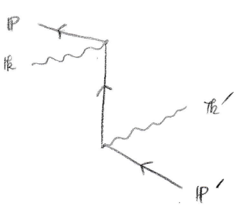
$|i\rangle = c^\dagger(p)a^\dagger(k)|0\rangle$ 、 $|f\rangle = c^\dagger(p')a^\dagger(k')|0\rangle$

$$\langle f|S_{B2}^{(2)}|i\rangle = -e^2\int d^4x_1\int d^4x_2 \left[\sqrt{\frac{m}{VE_{p'}}}\,\bar u(p')e^{ip'x_1}\right]\left[\sqrt{\frac{1}{2V\omega_{k'}}}\,\varepsilon_\nu(k')e^{ik'x_2}\right]\gamma^\mu\left[\frac{1}{(2\pi)^4}\int d^4q\, e^{-iq(x_1-x_2)}\, i\tilde S_F(q)\right]\gamma^\nu$$
$$\left[\sqrt{\frac{1}{2V\omega_k}}\,\varepsilon_\mu(k)e^{-ikx_1}\right]\left[\sqrt{\frac{m}{VE_p}}\,u(p)e^{-ipx_2}\right]$$
$$= \sqrt{\frac{m}{VE_{p'}}}\sqrt{\frac{1}{2V\omega_{k'}}}\sqrt{\frac{1}{2V\omega_k}}\sqrt{\frac{m}{VE_p}}\ \underbrace{(-e^2)\,\bar u(p')\not\varepsilon(k)\,i\tilde S_F(p-k')\,\not\varepsilon(k')u(p)}_{\mathcal{M}_{B2}^{(2)}}\cdot(2\pi)^4\delta^4(p'+k'-p-k)$$

$\uparrow$
$q = p'-k = p-k'$

• e^+のコンプトン散乱 B3 (Eq. 7.39)

$S_{B3}^{(2)}$



(Fig. 7.13a)

$$S_{B3}^{(2)} = -e^2\int d^4x_1\int d^4x_2\, N\left[(\bar\Psi^+ A_\mu^+)_{x_1}\gamma^\mu iS_F(x_1-x_2)\gamma^\nu(A_\nu^-\Psi^-)_{x_2}\right]$$
$$= -e^2\int d^4x_1\int d^4x_2\, (-1)\,\Psi_\beta^-(x_2)A_\nu^-(x_2)\left(\gamma^\mu iS_F(x_1-x_2)\gamma^\nu\right)_{\alpha\beta}A_\nu^+(x_1)\bar\Psi_\alpha^+(x_1)$$

$\uparrow$ スピノルの添字

$|i\rangle = d^\dagger(p)a^\dagger(k)|0\rangle$ 、 $|f\rangle = d^\dagger(p')a^\dagger(k')|0\rangle$

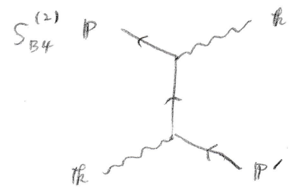
$$\langle f|S^{(2)}_{B3}|i\rangle = +e^2\int d^4x_1\int d^4x_2\left[\sqrt{\frac{m}{VE_{p'}}}\,v_\beta(p')e^{ip'x_2}\right]\left[\sqrt{\frac{1}{2V\omega_{k'}}}\,\varepsilon_\nu(k')e^{ik'x_2}\right]\left(\gamma^\mu\left[\frac{1}{(2\pi)^4}\int d^4q\, e^{-iq(x_1-x_2)}\,i\tilde{S}_F(q)\right]\gamma^\nu\right)_{\alpha\beta}$$

$$\hookrightarrow\left[\sqrt{\frac{1}{2V\omega_k}}\,\varepsilon_\mu(k)e^{-ikx_1}\right]\left[\sqrt{\frac{m}{VE_p}}\,\bar{v}_\alpha(p)e^{-ipx_1}\right]$$

$$=\sqrt{\frac{m}{VE_{p'}}}\sqrt{\frac{1}{2V\omega_{k'}}}\sqrt{\frac{1}{2V\omega_k}}\sqrt{\frac{m}{VE_p}}\;\underbrace{e^2\,\bar{v}(p)\,\not\varepsilon(k)\,i\tilde{S}_F(-p-k)\,\not\varepsilon(k')\,v(p')}_{\mathcal{M}^{(2)}_{B3}}\cdot(2\pi)^4\delta^4(p'+k'-p-k)$$

$\uparrow$ $q=-p-k=-p'-k'$

・e^+のコンプトン散乱 B4



(Fig. 7.13b)

$$S^{(2)}_{B4} = -e^2\int d^4x_1\int d^4x_2\, N\left[(\bar{\psi}^+A^-_\mu)_{x_1}\gamma^\mu iS_F(x_1-x_2)\gamma^\nu(A^+_\nu\psi^-)_{x_2}\right]$$

$$= -e^2\int d^4x_1\int d^4x_2\,(-1)\,\psi^-_\beta(x_2)A^-_\mu(x_1)\left(\gamma^\mu iS_F(x_1-x_2)\gamma^\nu\right)_{\alpha\beta}A^+_\nu(x_2)\bar{\psi}^+_\alpha(x_1)$$

$|i\rangle = d^+(p)a^+(k)|0\rangle$, $|f\rangle = d^+(p')a^+(k')|0\rangle$

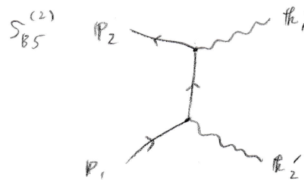
$$\langle f|S^{(2)}_{B4}|i\rangle = +e^2\int d^4x_1\int d^4x_2\left[\sqrt{\frac{m}{VE_{p'}}}\,v_\beta(p')e^{ip'x_2}\right]\left[\sqrt{\frac{1}{2V\omega_{k'}}}\,\varepsilon_\mu(k')e^{ik'x_1}\right]\left(\gamma^\mu\left[\frac{1}{(2\pi)^4}\int d^4q\, e^{-iq(x_1-x_2)}\,i\tilde{S}_F(q)\right]\gamma^\nu\right)_{\alpha\beta}$$

$$\hookrightarrow\left[\sqrt{\frac{1}{2V\omega_k}}\,\varepsilon_\nu(k)e^{-ikx_2}\right]\left[\sqrt{\frac{m}{VE_p}}\,\bar{v}_\alpha(p)e^{-ipx_1}\right]$$

$$=\sqrt{\frac{m}{VE_{p'}}}\sqrt{\frac{1}{2V\omega_{k'}}}\sqrt{\frac{1}{2V\omega_k}}\sqrt{\frac{m}{VE_p}}\;\underbrace{e^2\,\bar{v}(p)\,\not\varepsilon(k')\,i\tilde{S}_F(-p+k')\,\not\varepsilon(k)\,v(p')}_{\mathcal{M}^{(2)}_{B4}}\cdot(2\pi)^4\delta^4(p'+k'-p-k)$$

$\uparrow$ $q=-p+k'=-p'+k$

・e^+e^-対消滅 B5



$$S^{(2)}_{B5} = -e^2\int d^4x_1\int d^4x_2\, N\left[(\bar{\psi}^+A^-_\mu)_{x_1}\gamma^\mu iS_F(x_1-x_2)\gamma^\nu(A^-_\nu\psi^+)_{x_2}\right]$$

$$= -e^2\int d^4x_1\int d^4x_2\, A^-_\mu(x_1)\left(\gamma^\mu iS_F(x_1-x_2)\gamma^\nu\right)_{\alpha\beta}A^-_\nu(x_2)\bar{\psi}^+_\alpha(x_1)\psi^+_\beta(x_2)$$

$|i\rangle = c^+(p_1)d^+(p_2)|0\rangle$, $|f\rangle = a^+(k_1')a^+(k_2')|0\rangle$

$$\langle f|S^{(2)}_{B5}|i\rangle = -e^2\int d^4x_1\int d^4x_2\left[\sqrt{\frac{1}{2V\omega_{k_1'}}}\,\varepsilon_\mu(k_1')e^{ik_1'x_1}\right]\left[\sqrt{\frac{1}{2V\omega_{k_2'}}}\,\varepsilon_\nu(k_2')e^{ik_2'x_2}\right]\left(\gamma^\mu\left[\frac{1}{(2\pi)^4}\int d^4q\, e^{-iq(x_1-x_2)}\,i\tilde{S}_F(q)\right]\gamma^\nu\right)_{\alpha\beta}$$

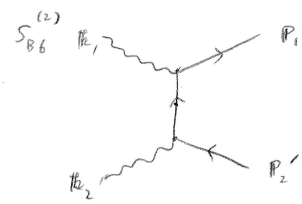
$$\hookrightarrow\left[\sqrt{\frac{1}{VE_{p_2}}}\,\bar{v}_\alpha(p_2)e^{-ip_2x_1}\right]\left[\sqrt{\frac{m}{VE_{p_1}}}\,u_\beta(p_1)e^{-ip_1x_2}\right]$$

$$=\sqrt{\frac{1}{2V\omega_{k_1'}}}\sqrt{\frac{1}{2V\omega_{k_2'}}}\sqrt{\frac{m}{VE_{p_1}}}\sqrt{\frac{m}{VE_{p_2}}}\;\underbrace{(-e^2)\,\bar{v}(p_2)\,\not\varepsilon(k_1')\,i\tilde{S}_F(p_1-k_2')\,\not\varepsilon(k_2')\,u(p_1)}_{\mathcal{M}^{(2)}_{B5}}\cdot(2\pi)^4\delta^4(k_1'+k_2'-p_1-p_2)$$

$\uparrow$ $q=-p_2+k_1'=p_1-k_2'$

※同種粒子があるときは経路に注意。

・e^+e^-対生成 B6



$$S^{(2)}_{B6} = -e^2\int d^4x_1\int d^4x_2\, N\left[(\bar{\psi}^-A^+_\mu)_{x_1}\gamma^\mu iS_F(x_1-x_2)\gamma^\nu(A^+_\nu\psi^-)_{x_2}\right]$$

$$= -e^2\int d^4x_1\int d^4x_2\,\bar{\psi}^-_\alpha(x_1)\psi^-_\beta(x_2)A^+_\mu(x_1)\left(\gamma^\mu iS_F(x_1-x_2)\gamma^\nu\right)_{\alpha\beta}A^+_\nu(x_2)$$

$|i\rangle = a^+(k_2)a^+(k_1)|0\rangle$, $|f\rangle = c^+(p_1')d^+(p_2')|0\rangle$

$$\langle f|S^{(2)}_{B6}|i\rangle = -e^2\int d^4x_1\int d^4x_2\left[\sqrt{\frac{m}{VE_{p_1'}}}\,\bar{u}_\alpha(p_1')e^{ip_1'x_1}\right]\left[\sqrt{\frac{m}{VE_{p_2'}}}\,v_\beta(p_2')e^{ip_2'x_2}\right]\left(\gamma^\mu\left[\frac{1}{(2\pi)^4}\int d^4q\, e^{-iq(x_1-x_2)}\,i\tilde{S}_F(q)\right]\gamma^\nu\right)_{\alpha\beta}$$

$$\hookrightarrow\left[\sqrt{\frac{1}{2V\omega_{k_1}}}\,\varepsilon_\mu(k_1)e^{-ik_1x_1}\right]\left[\sqrt{\frac{1}{2V\omega_{k_2}}}\,\varepsilon_\nu(k_2)e^{-ik_2x_2}\right]$$

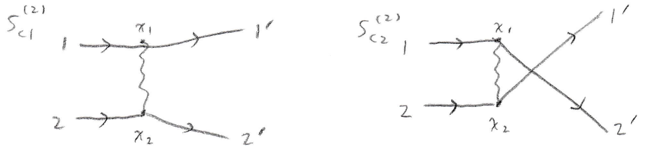
$$=\sqrt{\frac{m}{VE_{p_1'}}}\sqrt{\frac{m}{VE_{p_2'}}}\sqrt{\frac{1}{2V\omega_{k_1}}}\sqrt{\frac{1}{2V\omega_{k_2}}}\;\underbrace{(-e^2)\,\bar{u}(p_1')\,\not\varepsilon(k_1)\,i\tilde{S}_F(p_1'-k_1)\,\not\varepsilon(k_2)\,v(p_2')}_{\mathcal{M}^{(2)}_{B6}}\cdot(2\pi)^4\delta^4(p_1'+p_2'-k_1-k_2)$$

$\uparrow$ $q=p_1'-k_1=-p_2'+k_2$

7.2.3 電子-電子散乱

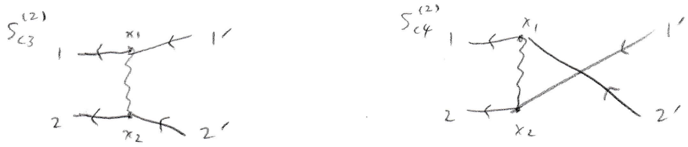
$S_c^{(2)}$ (式(7.5c)) から生じる散乱過程には、以下の6種がある。

e^- のメラー散乱



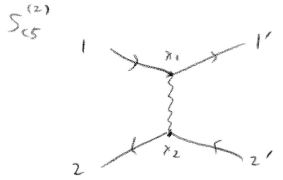
$\bar{\Psi}^- \gamma^\mu \Psi^+ iD_{F\mu\nu} \bar{\Psi}^- \gamma^\nu \Psi^+$

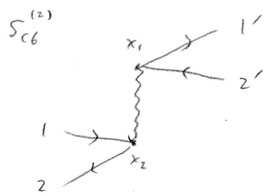
e^+ のメラー散乱



$\bar{\Psi}^+ \gamma^\mu \Psi^- iD_{F\mu\nu} \bar{\Psi}^+ \gamma^\nu \Psi^-$

バーバー散乱



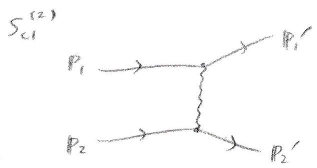


$(\bar{\Psi}^+ \gamma^\nu \Psi^- iD_{F\mu\nu} \bar{\Psi}^- \gamma^\mu \Psi^+) + (\bar{\Psi}^- \gamma^\mu \Psi^+ iD_{F\mu\nu} \bar{\Psi}^+ \gamma^\nu \Psi^-)$

$(\bar{\Psi}^- \gamma^\mu \Psi^- iD_{F\mu\nu} \bar{\Psi}^+ \gamma^\nu \Psi^+) + (\bar{\Psi}^+ \gamma^\mu \Psi^+ iD_{F\mu\nu} \bar{\Psi}^- \gamma^\nu \Psi^-)$

※ $S_c^{(2)} = -\frac{e^2}{2}\int d^4x_1 \int d^4x_2 \, N\left[(\cdots)_{x_1} iD_{F\mu\nu}(x_1-x_2)(\cdots)_{x_2}\right]$ の [ ] 内を、引数を省略して書く。

・e^- のメラー散乱 C1 (Eq. 7.41b)



(Fig. 7.14a)

$$
\begin{aligned}
S_{c1}^{(2)} &= -e^2 \int d^4x_1 \int d^4x_2 \, N\left[(\bar{\Psi}_{1'}^- \gamma^\mu \Psi_1^+)_{x_1} (\bar{\Psi}_{2'}^- \gamma^\nu \Psi_2^+)_{x_2}\right] iD_{F\mu\nu}(x_1-x_2) \\
&= -e^2 \int d^4x_1 \int d^4x_2 \, (-1)^2 \bar{\Psi}_{2'\delta}^-(x_2) \bar{\Psi}_{1'\alpha}^-(x_1) \gamma^\mu_{\alpha\beta} iD_{F\mu\nu}(x_1-x_2) \gamma^\nu_{\delta\varepsilon} \Psi_{1\beta}^+(x_1) \Psi_{2\varepsilon}^+(x_2)
\end{aligned}
$$

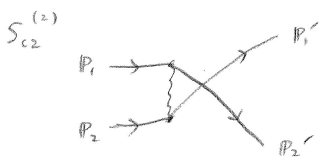
$|i\rangle = c^\dagger(p_2) c^\dagger(p_1)|0\rangle$ 、 $|f\rangle = c^\dagger(p_2') c^\dagger(p_1')|0\rangle$

$$
\begin{aligned}
\langle f | S_{c1}^{(2)} | i \rangle &= -e^2 \int d^4x_1 \int d^4x_2 \left[\sqrt{\frac{m}{VE_{p_2'}}} \bar{u}_\delta(p_2') e^{ip_2'x_2}\right] \left[\sqrt{\frac{m}{VE_{p_1'}}} \bar{u}_\alpha(p_1') e^{ip_1'x_1}\right] \gamma^\mu_{\alpha\beta} \left[\frac{1}{(2\pi)^4} \int d^4k \, e^{-ik(x_1-x_2)} i\tilde{D}_{F\mu\nu}(k)\right] \gamma^\nu_{\delta\varepsilon} \\
&\qquad \left[\sqrt{\frac{m}{VE_{p_1}}} u_\beta(p_1) e^{-ip_1x_1}\right] \left[\sqrt{\frac{m}{VE_{p_2}}} u_\varepsilon(p_2) e^{-ip_2x_2}\right] \\
&= \sqrt{\frac{m}{VE_{p_1'}}} \sqrt{\frac{m}{VE_{p_2'}}} \sqrt{\frac{m}{VE_{p_1}}} \sqrt{\frac{m}{VE_{p_2}}} \underbrace{(-e^2) \bar{u}(p_1') \gamma^\mu u(p_1) i\tilde{D}_{F\mu\nu}(p_2-p_2') \bar{u}(p_2') \gamma^\nu u(p_2)}_{\mathcal{M}_{c1}^{(2)}} \cdot (2\pi)^4 \delta^4(p_1'+p_2'-p_1-p_2)
\end{aligned}
$$

$\uparrow$

$k = -p_1 + p_1' = p_2 - p_2'$

・e^- のメラー散乱 C2 (Eq. 7.41c)



(Fig. 7.14b)

$$
\begin{aligned}
S_{c2}^{(2)} &= -e^2 \int d^4x_1 \int d^4x_2 \, N\left[(\bar{\Psi}_{2'}^- \gamma^\mu \Psi_1^+)_{x_1} (\bar{\Psi}_{1'}^- \gamma^\nu \Psi_2^+)_{x_2}\right] iD_{F\mu\nu}(x_1-x_2) \\
&= -e^2 \int d^4x_1 \int d^4x_2 \, (-1) \bar{\Psi}_{2'\alpha}^-(x_1) \bar{\Psi}_{1'\delta}^-(x_2) \gamma^\mu_{\alpha\beta} iD_{F\mu\nu}(x_1-x_2) \gamma^\nu_{\delta\varepsilon} \Psi_{1\beta}^+(x_1) \Psi_{2\varepsilon}^+(x_2)
\end{aligned}
$$

$|i\rangle = c^\dagger(p_2) c^\dagger(p_1)|0\rangle$ 、 $|f\rangle = c^\dagger(p_2') c^\dagger(p_1')|0\rangle$

$$
\begin{aligned}
\langle f | S_{c2}^{(2)} | i \rangle &= +e^2 \int d^4x_1 \int d^4x_2 \left[\sqrt{\frac{m}{VE_{p_2'}}} \bar{u}_\alpha(p_2') e^{ip_2'x_1}\right] \left[\sqrt{\frac{m}{VE_{p_1'}}} \bar{u}_\delta(p_1') e^{ip_1'x_2}\right] \gamma^\mu_{\alpha\beta} \left[\frac{1}{(2\pi)^4} \int d^4k \, e^{-ik(x_1-x_2)} i\tilde{D}_{F\mu\nu}(k)\right] \gamma^\nu_{\delta\varepsilon} \\
&\qquad \left[\sqrt{\frac{m}{VE_{p_1}}} u_\beta(p_1) e^{-ip_1x_1}\right] \left[\sqrt{\frac{m}{VE_{p_2}}} u_\varepsilon(p_2) e^{-ip_2x_2}\right] \\
&= \sqrt{\frac{m}{VE_{p_1'}}} \sqrt{\frac{m}{VE_{p_2'}}} \sqrt{\frac{m}{VE_{p_1}}} \sqrt{\frac{m}{VE_{p_2}}} \underbrace{e^2 \bar{u}(p_2') \gamma^\mu u(p_1) i\tilde{D}_{F\mu\nu}(p_2-p_1') \bar{u}(p_1') \gamma^\nu u(p_2)}_{\mathcal{M}_{c2}^{(2)}} \cdot (2\pi)^4 \delta^4(p_1'+p_2'-p_1-p_2)
\end{aligned}
$$

$\uparrow$

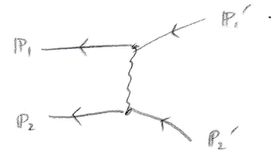
$k = -p_1 + p_2' = -p_1' + p_2$

※ $|i\rangle = c^\dagger(p_2) c^\dagger(p_1)|0\rangle = -c^\dagger(p_1) c^\dagger(p_2)|0\rangle$ 等なので、$\mathcal{M}_{c1}^{(2)}$, $\mathcal{M}_{c2}^{(2)}$ は必ず符号が負, 正というわけではない。

ただし、共通の $|i\rangle$, $|f\rangle$ を用いたとき、$\mathcal{M}_{c1}^{(2)}$ と $\mathcal{M}_{c2}^{(2)}$ の符号は逆になることは決まっている。

※ $\tilde{D}_{F\mu\nu}(k)$ は偶関数なので、内線の光子はどちら向きを選んでもよい。

・e^+ のメラー散乱 C3

$S_{c3}^{(2)}$ 

$$S_{c3}^{(2)} = -e^2\int d^4x_1\int d^4x_2\, N\left[(\bar{\psi}_1^+\gamma^\mu\psi_{1'}^-)_{x_1}(\bar{\psi}_2^+\gamma^\nu\psi_{2'}^-)_{x_2}\right] iD_{F\mu\nu}(x_1-x_2)$$

$$= -e^2\int d^4x_1\int d^4x_2\,(-1)^4\,\psi_{2'\varepsilon}^-(x_2)\,\psi_{1'\beta}^-(x_1)\,\gamma^\mu_{\alpha\beta}\,iD_{F\mu\nu}(x_1-x_2)\,\gamma^\nu_{\delta\varepsilon}\,\bar{\psi}_{1\alpha}^+(x_1)\,\bar{\psi}_{2\delta}^+(x_2)$$

$|i\rangle = d^+(p_2)d^+(p_1)|0\rangle$ 、 $|f\rangle = d^+(p_2')d^+(p_1')|0\rangle$

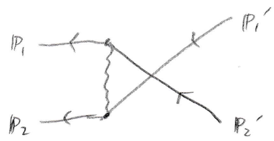
$$\langle f|S_{c3}^{(2)}|i\rangle = -e^2\int d^4x_1\int d^4x_2\left[\sqrt{\frac{m}{VE_{p_2'}}}v_\varepsilon(p_2')e^{ip_2'x_2}\right]\left[\sqrt{\frac{m}{VE_{p_1'}}}v_\beta(p_1')e^{ip_1'x_1}\right]\gamma^\mu_{\alpha\beta}\left[\frac{1}{(2\pi)^4}\int d^4k\,e^{-ik(x_1-x_2)}\,i\tilde{D}_{F\mu\nu}(k)\right]\gamma^\nu_{\delta\varepsilon}$$

$$\times\left[\sqrt{\frac{m}{VE_{p_1}}}\bar{v}_\alpha(p_1)e^{-ip_1x_1}\right]\left[\sqrt{\frac{m}{VE_{p_2}}}\bar{v}_\delta(p_2)e^{-ip_2x_2}\right]$$

$$= \sqrt{\frac{m}{VE_{p_1'}}}\sqrt{\frac{m}{VE_{p_2'}}}\sqrt{\frac{m}{VE_{p_1}}}\sqrt{\frac{m}{VE_{p_2}}}\underbrace{(-e^2)\bar{v}(p_1)\gamma^\mu v(p_1')\,i\tilde{D}_{F\mu\nu}(p_2-p_2')\,\bar{v}(p_2)\gamma^\nu v(p_2')}_{\mathcal{M}_{c3}^{(2)}}\cdot(2\pi)^4\delta^4(p_1'+p_2'-p_1-p_2)$$

$k = -p_1+p_1' = p_2-p_2'$

・e^+ のメラー散乱 C4

$S_{c4}^{(2)}$ 

$$S_{c4}^{(2)} = -e^2\int d^4x_1\int d^4x_2\, N\left[(\bar{\psi}_1^+\gamma^\mu\psi_{2'}^-)_{x_1}(\bar{\psi}_2^+\gamma^\nu\psi_{1'}^-)_{x_2}\right] iD_{F\mu\nu}(x_1-x_2)$$

$$= -e^2\int d^4x_1\int d^4x_2\,(-1)^3\,\psi_{2'\beta}^-(x_1)\,\psi_{1'\varepsilon}^-(x_2)\,\gamma^\mu_{\alpha\beta}\,iD_{F\mu\nu}(x_1-x_2)\,\gamma^\nu_{\delta\varepsilon}\,\bar{\psi}_{1\alpha}^+(x_1)\,\bar{\psi}_{2\delta}^+(x_2)$$

$|i\rangle = d^+(p_2)d^+(p_1)|0\rangle$ 、 $|f\rangle = d^+(p_2')d^+(p_1')|0\rangle$

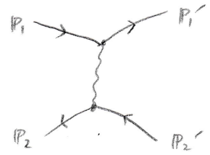
$$\langle f|S_{c4}^{(2)}|i\rangle = +e^2\int d^4x_1\int d^4x_2\left[\sqrt{\frac{m}{VE_{p_2'}}}v_\beta(p_2')e^{ip_2'x_1}\right]\left[\sqrt{\frac{m}{VE_{p_1'}}}v_\varepsilon(p_1')e^{ip_1'x_2}\right]\gamma^\mu_{\alpha\beta}\left[\frac{1}{(2\pi)^4}\int d^4k\,e^{-ik(x_1-x_2)}\,i\tilde{D}_{F\mu\nu}(k)\right]\gamma^\nu_{\delta\varepsilon}$$

$$\times\left[\sqrt{\frac{m}{VE_{p_1}}}\bar{v}_\alpha(p_1)e^{-ip_1x_1}\right]\left[\sqrt{\frac{m}{VE_{p_2}}}\bar{v}_\delta(p_2)e^{-ip_2x_2}\right]$$

$$= \sqrt{\frac{m}{VE_{p_1'}}}\sqrt{\frac{m}{VE_{p_2'}}}\sqrt{\frac{m}{VE_{p_1}}}\sqrt{\frac{m}{VE_{p_2}}}\underbrace{e^2\bar{v}(p_1)\gamma^\mu v(p_2')\,i\tilde{D}_{F\mu\nu}(p_2-p_1')\,\bar{v}(p_2)\gamma^\nu v(p_1')}_{\mathcal{M}_{c4}^{(2)}}\cdot(2\pi)^4\delta^4(p_1'+p_2'-p_1-p_2)$$

$k = -p_1+p_2' = -p_1'+p_2$

・バーバー散乱 C5　　(問7.1)

$S_{c5}^{(2)}$ 

$$S_{c5}^{(2)} = -e^2\int d^4x_1\int d^4x_2\, N\left[(\bar{\psi}_{1'}^-\gamma^\mu\psi_1^+)_{x_1}(\bar{\psi}_2^+\gamma^\nu\psi_{2'}^-)_{x_2}\right] iD_{F\mu\nu}(x_1-x_2)$$

$$= -e^2\int d^4x_1\int d^4x_2\,(-1)^3\,\psi_{2'\varepsilon}^-(x_2)\,\bar{\psi}_{1'\alpha}^-(x_1)\,\gamma^\mu_{\alpha\beta}\,iD_{F\mu\nu}(x_1-x_2)\,\gamma^\nu_{\delta\varepsilon}\,\psi_{1\beta}^+(x_1)\,\bar{\psi}_{2\delta}^+(x_2)$$

$|i\rangle = d^+(p_2)c^+(p_1)|0\rangle$ 、 $|f\rangle = d^+(p_2')c^+(p_1')|0\rangle$

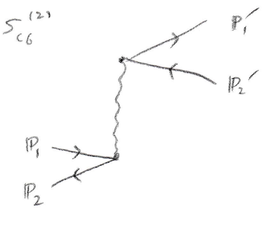
$$\langle f|S_{c5}^{(2)}|i\rangle = +e^2\int d^4x_1\int d^4x_2\left[\sqrt{\frac{m}{VE_{p_2'}}}v_\varepsilon(p_2')e^{ip_2'x_2}\right]\left[\sqrt{\frac{m}{VE_{p_1'}}}\bar{u}_\alpha(p_1')e^{ip_1'x_1}\right]\gamma^\mu_{\alpha\beta}\left[\frac{1}{(2\pi)^4}\int d^4k\,e^{-ik(x_1-x_2)}\,i\tilde{D}_{F\mu\nu}(k)\right]\gamma^\nu_{\delta\varepsilon}$$

$$\times\left[\sqrt{\frac{m}{VE_{p_1}}}u_\beta(p_1)e^{-ip_1x_1}\right]\left[\sqrt{\frac{m}{VE_{p_2}}}\bar{v}_\delta(p_2)e^{-ip_2x_2}\right]$$

$$= \sqrt{\frac{m}{VE_{p_1'}}}\sqrt{\frac{m}{VE_{p_2'}}}\sqrt{\frac{m}{VE_{p_1}}}\sqrt{\frac{m}{VE_{p_2}}}\underbrace{e^2\bar{u}(p_1')\gamma^\mu u(p_1)\,i\tilde{D}_{F\mu\nu}(p_2-p_2')\,\bar{v}(p_2)\gamma^\nu v(p_2')}_{\mathcal{M}_{c5}^{(2)}}\cdot(2\pi)^4\delta^4(p_1'+p_2'-p_1-p_2)$$

$k = -p_1+p_1' = p_2-p_2'$

・バーバー散乱 C6　　(問7.1)

$S_{c6}^{(2)}$ 

$$S_{c6}^{(2)} = -e^2\int d^4x_1\int d^4x_2\, N\left[(\bar{\psi}_{1'}^-\gamma^\mu\psi_{2'}^-)_{x_1}(\bar{\psi}_2^+\gamma^\nu\psi_1^+)_{x_2}\right] iD_{F\mu\nu}(x_1-x_2)$$

$$= -e^2\int d^4x_1\int d^4x_2\,(-1)^2\,\psi_{2'\beta}^-(x_1)\,\bar{\psi}_{1'\alpha}^-(x_1)\,\gamma^\mu_{\alpha\beta}\,iD_{F\mu\nu}(x_1-x_2)\,\gamma^\nu_{\delta\varepsilon}\,\psi_{1\varepsilon}^+(x_2)\,\bar{\psi}_{2\delta}^+(x_2)$$

$|i\rangle = d^+(p_2)c^+(p_1)|0\rangle$ 、 $|f\rangle = d^+(p_2')c^+(p_1')|0\rangle$

$$\langle f | S_{c6}^{(2)} | i \rangle = -e^2 \int d^4x_1 \int d^4x_2 \left[\sqrt{\frac{m}{VE_{p_2'}}} v_\beta(p_2') e^{ip_2'x_1} \right] \left[\sqrt{\frac{m}{VE_{p_1'}}} \bar{u}_\alpha(p_1') e^{ip_1'x_1} \right] \gamma^\mu_{\alpha\beta} \left[\frac{1}{(2\pi)^4} \int d^4k \, e^{-ik(x_1-x_2)} \, i D_{F\mu\nu}(k) \right] \gamma^\nu_{\delta\varepsilon} \, \lrcorner$$

$$\hookrightarrow \left[\sqrt{\frac{m}{VE_{p_1}}} u_\varepsilon(p_1) e^{-ip_1x_2} \right] \left[\sqrt{\frac{m}{VE_{p_2}}} \bar{v}_\delta(p_2) e^{-ip_2x_2} \right]$$

$$= \sqrt{\frac{m}{VE_{p_1'}}} \sqrt{\frac{m}{VE_{p_2'}}} \sqrt{\frac{m}{VE_{p_1}}} \sqrt{\frac{m}{VE_{p_2}}} \underbrace{(-e^2) \bar{u}(p_1') \gamma^\mu v(p_2') \, i \tilde{D}_{F\mu\nu}(p_1+p_2) \bar{v}(p_2) \gamma^\nu u(p_1)}_{\mathcal{M}_{c6}^{(2)}} \cdot (2\pi)^4 \delta^4(p_1'+p_2'-p_1-p_2)$$

$$k = p_1' + p_2' = p_1 + p_2$$