Srednicki §8 THE PATH INTEGRAL FOR FREE FIELD THEORY

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- §7で行った内容を自由場で再び行う
- 途中 Feynman伝播関数を定義し、ウィックの定理を得る

harmonic oscillator のハミルトニアン密度は自由場に一般化すると

$$\mathcal{H}_0 = \frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}m^2\varphi^2 \qquad (8.1)$$

§7との対応は

$$\begin{aligned} q(t) &\rightarrow \varphi(\mathbf{x},t) && \text{(classical field)} \\ Q(t) &\rightarrow \varphi(\mathbf{x},t) && \text{(operator field)} \\ f(t) &\rightarrow J(\mathbf{x},t) && \text{(classical source)} \end{aligned} \qquad (8.2)$$

文脈で区別

ここで $\mathcal{H}_0 \rightarrow (1-i\varepsilon)\mathcal{H}_0$ の操作は
$\mathcal{H}_0$中の $m^2 \rightarrow (m^2 - i\varepsilon)$ と結果が等しく、
以後 $m^2 - i\varepsilon$ を m^2と書く

経路積分は、

$$Z_0(J) = \langle 0|0\rangle_J = \int \mathcal{D}\varphi\ e^{i\int d^4x[\mathcal{L}_0 + J\varphi]} \qquad (8.3)$$

但し、

$$\mathcal{L}_0 = -\frac{1}{2}\partial^\mu\varphi\,\partial_\mu\varphi - \frac{1}{2}m^2\varphi^2 \quad (8.4), \qquad \mathcal{D}\varphi \propto \prod_x d\varphi(x) \quad (8.5)$$

§7と同様にZ₀(J)は評価できる、最初に4次元フーリエ積分を定義する

$$\tilde{\varphi}(k) = \int d^4x\, e^{-ikx}\varphi(x), \quad \varphi(x) = \int \frac{d^4k}{(2\pi)^4} e^{ikx}\tilde{\varphi}(k) \qquad (8.6)$$

但し、 $kx = -k^0t + \mathbf{k}\cdot\mathbf{x}$

$$S_0 = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left[-\tilde{\varphi}(k)(k^2+m^2)\tilde{\varphi}(-k)+\tilde{J}(k)\tilde{\varphi}(-k)+\tilde{J}(-k)\tilde{\varphi}(k)\right] \quad (8.7)$$

但し、 $k^2 = \boldsymbol{k}^2 - k_0^2$

ここで経路積分の変数を

$$\tilde{\chi}(k) = \tilde{\varphi}(k) - \frac{\tilde{J}(k)}{k^2+m^2}$$ とすることを考える (8.8)

定数によりシフトする変換なので

$$D\varphi = D\chi$$

すると (8.7) は次のようにできる

$$(8.7) = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left[\frac{\tilde{J}(k)\tilde{J}(-k)}{k^2+m^2} - \tilde{\chi}(k)(k^2+m^2)\tilde{\chi}(-k)\right] \quad (8.9)$$

$$Z_0(J) = \int D\chi \exp\left[\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\frac{\tilde{J}(k)\tilde{J}(-k)}{k^2+m^2}\right] \cdot \exp\left[\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\left\{-\tilde{\chi}(k)\tilde{\chi}(-k)(k^2+m^2)\right\}\right]$$

$$= \exp\left[\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\frac{\tilde{J}(k)\tilde{J}(-k)}{k^2+m^2}\right]\cdot \underbrace{\int D\chi \exp\left[\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\left\{-\tilde{\chi}(k)\tilde{\chi}(-k)(k^2+m^2)\right\}\right]}_{=1}$$

$$\left[\begin{array}{l} \tilde{J}(k) = \int d^4x \; e^{ikx} J(x,t) \\ \tilde{J}(-k) = \int d^4x' \; e^{-ikx'} J(x',t) \\ \qquad\qquad\qquad\qquad\qquad より \end{array}\right]$$

$$= \exp\left[\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\int d^4x \int d^4x' \; \frac{e^{ik(x-x')}}{k^2+m^2-i\varepsilon} \; J(x)J(x')\right]$$

$$= \exp\left[\frac{i}{2}\int d^4x \, d^4x' \; J(x)\,\Delta(x-x')\,J(x')\right] \qquad (8.10)$$

但し、$\Delta(x-x')$ は Feynman propagator (伝播函数)

$$\Delta(x-x') = \int\frac{d^4k}{(2\pi)^4}\;\frac{e^{ik(x-x')}}{k^2+m^2-i\varepsilon} \qquad (8.11)$$

これは Klein-Gordon eq の Green's function.

$$(-\partial_x^2+m^2)\,\Delta(x-x') = \delta^4(x-x') \qquad (8.12)$$

$$\left[\begin{array}{l} \because \\ \lim_{\varepsilon\to 0} 左辺 = \int\frac{d^4k}{(2\pi)^4}\;\frac{k^2+m^2}{k^2+m^2}\,e^{ik(x-x')} \\ \qquad\qquad = \delta^4(x-x') \quad (= 右辺) \end{array}\right]$$

また次のようにも表すことができる

$$\Delta(x-x') = i\int \widetilde{dk} \;\; e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')-i\omega|t-t'|}$$

$$= i\theta(t-t')\int \widetilde{dk}\, e^{ik(x-x')} + i\theta(t'-t)\int \widetilde{dk}\, e^{-ik(x-x')}$$

(8.13)

但し、$\widetilde{dk} \equiv \dfrac{d^3k}{(2\pi)^3 2\omega}$ (3.18)

$\omega = (k^2 + m^2)^{\frac{1}{2}}$ (3.11)

θ は step function

(7.15) との類似性から

$$\langle 0|T\varphi(x_1)\varphi(x_2)\cdots|0\rangle = \frac{1}{i}\frac{\delta}{\delta J(x_1)} \cdots\cdots Z_0(J)\Big|_{J=0}$$

ex. 1

$$\langle 0|T\varphi(x_1)\varphi(x_2)|0\rangle = \frac{1}{i}\frac{\delta}{\delta J(x_1)} \cdot \frac{1}{i}\frac{\delta}{\delta J(x_2)} Z_0(J)\Big|_{J=0}$$

$$= \frac{1}{i}\frac{\delta}{\delta J(x_1)}\frac{1}{i}\left(\frac{\delta}{\delta J(x_2)} \exp\left[\frac{i}{2}\int dx^4 dx'^4\, J(x)J(x')\Delta(x-x')\right]\right)Z_0(J)\Big|_{J=0}$$

$$= \frac{1}{i}\frac{\delta}{\delta J(x_1)}\left\{\frac{1}{2}\int d^4x'\, J(x')\Delta(x_2-x') + \frac{1}{2}\int d^4x\, J(x)\Delta(x_2-x)\right\} Z_0(J)\Big|_{J=0}$$

$$= \frac{1}{i}\left[\frac{\delta}{\delta J(x_1)}\int d^4x\, J(x)\Delta(x_2-x)\right] Z_0(J)\Big|_{J=0}$$

$$= \frac{1}{i}\left[\Delta(x_2-x_1) + i\int d^4x'\, \Delta(x_1-x')J(x')\int d^4x'\, \Delta(x_2-x')J(x')\right]Z_0(J)\Big|_{J=0}$$

$$= \frac{1}{i}\Delta(x_2-x_1)$$

(8.15)

ex. 2

$$\langle 0|T\varphi(x_1)\varphi(x_2)\varphi(x_3)|0\rangle = 0$$

ex. 3

$$\langle 0|T\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)|0\rangle$$

$$= \frac{1}{i^2}\left[\Delta(x_1-x_2)\Delta(x_3-x_4)+\Delta(x_1-x_3)\Delta(x_2-x_4)+\Delta(x_1-x_4)\Delta(x_2-x_3)\right]$$

(8.16)

一般に nが奇数のとき $\langle 0|T\varphi(x_1)\cdots\varphi(x_n)|0\rangle = 0$

$$\boxed{\begin{aligned}&\langle 0|T\varphi(x_1)\cdots\varphi(x_{2n})|0\rangle \\ &= \frac{1}{i^n}\sum_{\text{pairings}}\Delta(x_{i_1}-x_{i_2})\cdots\Delta(x_{i_{2n-1}}-x_{i_{2n}})\end{aligned}}$$

(8.17)

Wick's theorem