Neut

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Contents
- Brief History ~ Requirements
- Neutrino interactions above ~100MeV
- Procedure of the neutrino event generation
- Used models for each neutrino interaction mode
- Nuclear effects (Final state interactions of hadrons)
- Remaining issues

0. Before starting this talk

ニュートリノー核子･原子核反応シミュレーションの重要性

大気ニュートリノ観測実験 + 核子崩壊探索実験
長基線ニュートリノ振動実験

ニュートリノの直接観測はできない

検出器内で発生した
ニュートリノー核子･原子核散乱
による生成粒子を観測する。

主として用いられる反応

Charged current interactions with nucleus

$\nu + N \rightarrow l + N' \ (+ X)$

N, N' : nucleus
l : charged lepton
X : hadrons (π, K, Ω, nucleons etc…)

→ ニュートリノー核子･原子核散乱の正確な理解が必要

1. Introduction

Example 1: 加速器を用いたニュートリノ実験の場合

ニュートリノの方向は既知

Case 1: Eν = 100 ~ 1 GeV

Select charged current quasi-elastic scattering events

$\nu + N \rightarrow l + N'$

生成した荷電レプトンの運動量及び方向を用いて

ニュートリノのエネルギーを再構成

反応種別の同定が必要

→ Selection efficiency や purity / background contamination

は、シミュレーションを用いて評価する。

Case 2: Eν > several GeV

Charged current deep inelastic scattering events dominate.

$\nu + N \rightarrow l + N' + \text{hadrons}$

ニュートリノエネルギーの再構成には

生成荷電レプトンの運動量と方向、ならびに

検出器において観測されたハドロンのエネルギーを用いる。

Precise knowledge of the primary neutrino interactions

and the secondary hadron interactions is important.

1. Introduction

Example 2: 大気ニュートリノ観測実験

大半の解析においては、
観測されたレプトンの運動量及び方向を用いる。

ニュートリノ振動パラメータは、
観測データと振動を考慮にいれたシミュレーションを
用いることで決定される。

Example 3: 核子崩壊探索実験

陽子崩壊の主要なモードと予言される$p \rightarrow e^+ \pi^0$では、
single π productionが主要なバックグラウンドとなる。
（終状態の粒子が同一である）

ニュートリノ反応や核子崩壊によって、
原子核中で生成したπ粒子などは、
原子核中で散乱・吸収されることもしられている。.
($\pi^+ n \rightarrow \pi^0 p$ etc.)

Precise knowledge of the primary neutrino interactions
and the secondary hadron interactions is important.

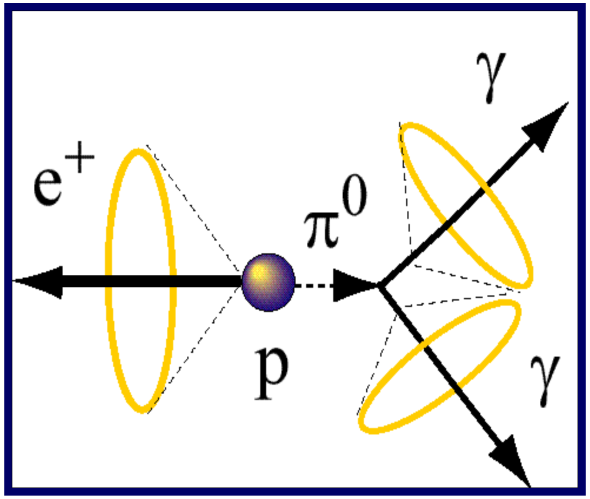
1. Brief History

Minimal SU(5) GUT predicts nucleon decay.

Major decay mode of proton is $p \rightarrow e^+ + \pi^0$

- Ring imaging water Cherenkov detector has high efficiency in reconstructing electrons and low momentum π^0s.

- Possible to build massive detector to search for the nucleon decay predicted by the theory.



But π^0 may interact in nucleus if the proton in Oxygen decays.

↓

Study of interaction of π in Oxygen is important for the analysis.

Need simulation program to determine the event selection criteria and to estimate the detection efficiency.

1. Brief History

Ring imaging water Cherenkov detectors

Japan 1983 : Kamiokande (3,000 ton, ~ 1,000 20inch PMTs)
1996 : Super-Kamiokande (50,000 ton, ~12,000 20inch PMTs)

*) Also in US, IMB experiment started in 1982.
(7,000 ton tank, ~ 2,000 5(8)inch PMTs)

Detect relativistic particles
running through the detector medium (water)
using the Cherenkov light.

- Number of the Cherenkov rings
  = Observed number of particles
- Energy (momentum) is reconstructed
  using the total charge of the ring.
- Direction is reconstructed
  using the image of the ring.
- Particle type is identified
  using the shape of the ring.
  - **Electron / gamma : Diffused**
  - **Muon etc. : Sharp**

Neut was initially developed for the Kamiokande experiment.

1. Brief History

The major background in searching for the nucleon decay
is caused by the atmospheric neutrinos.

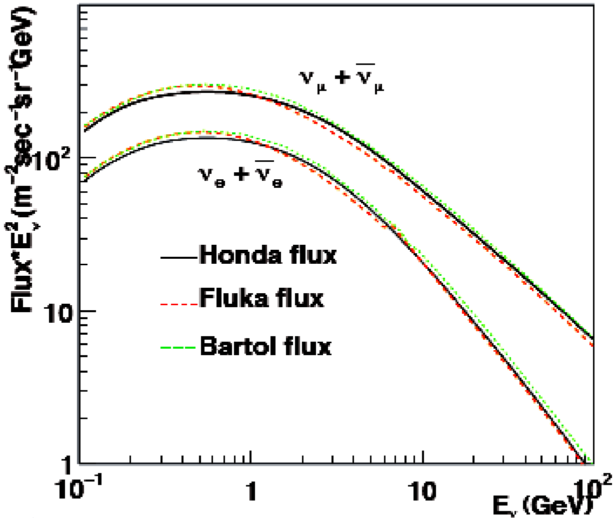
One of the neutrino interaction
in a few GeV region

Single pion production

$\overline{\nu}_e + p \rightarrow e^+ + n + \pi^0$

$\nu_e + n \rightarrow e^- + p + \pi^0$ etc..

Atmospheric neutrino flux



Basically, the electron (positron) and π^0
from primary neutrino interaction
have different energy/momentum
from nucleon decay products.

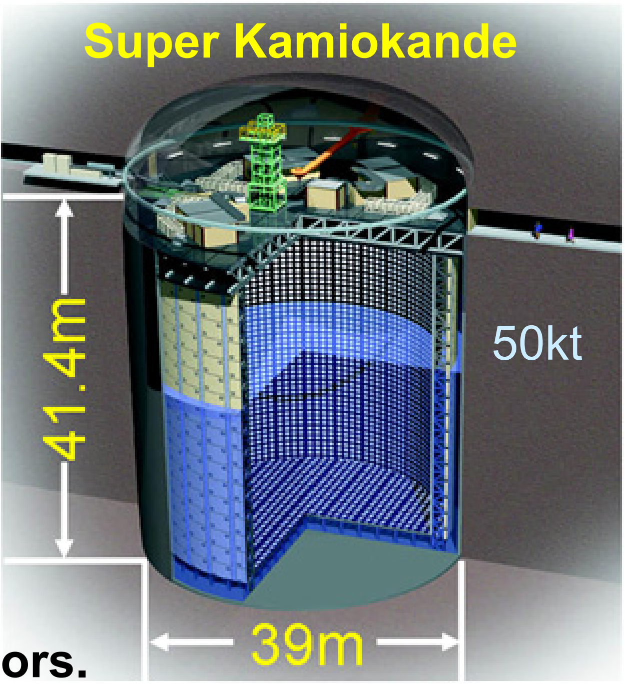
But re-scattering of π in Oxygen changes its charge or momentum.

→ ***Need precise simulation program to estimate the background.***

- Primary neutrino interaction with water
  (= neutrino interaction with Oxygen or proton)
- π re-scattering in Oxygen

1. Brief History

Indication of neutrino oscillation
from the observation of solar and atmospheric neutrino



- Super-Kamiokande

  Massive (50kt) Water Cherenkov detector

  ~30 times larger statistics
  High precision observation.

  1998 Evidence of neutrino oscillation

- Accelerator based neutrino experiment

  K2K ~ SciBooNE

Beam experiment ~ High statistics @ near detectors.

Several types of the detectors with various target material

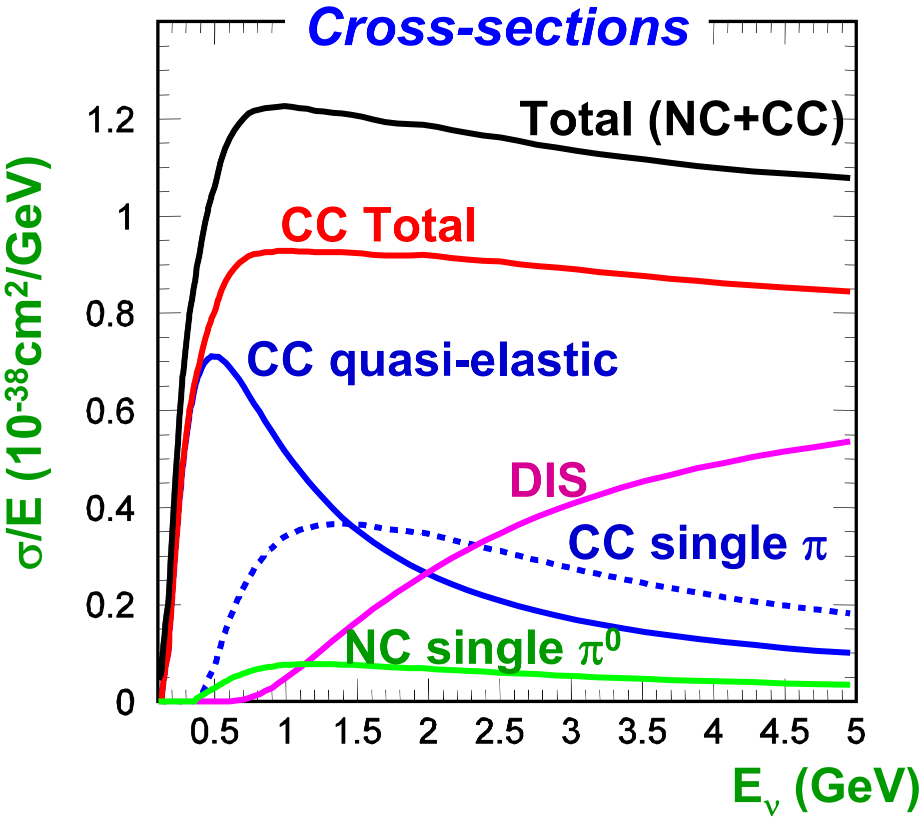
Water Chrenkov detector
Water target tracking detector
Scintillator detector
Iron target muon range detector

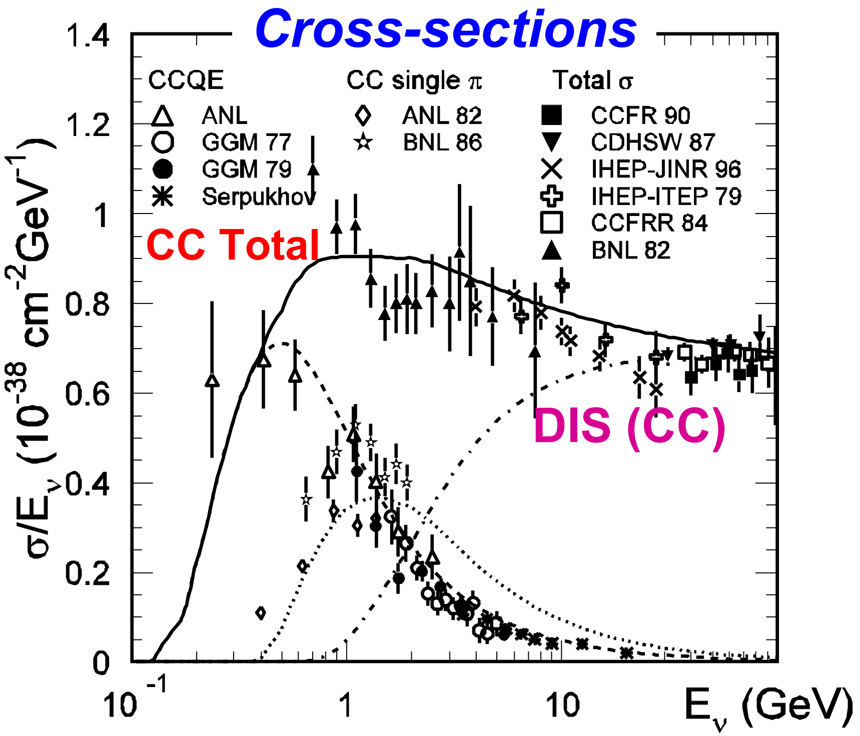
Need much more precise neutrino interaction simulation program.
New experimental data and models make various improvements possible.

2. Neutrino interactions

Charged current quasi-elastic scattering	$\nu_\mu + n \rightarrow \mu^- + p$
Neutral current elastic scattering	$\nu_\mu + N \rightarrow \nu + N$
Single π,η,K resonance productions	$\nu_\mu + N \rightarrow l + N' + \pi\ (\eta, K)$
Coherent pion productions	$\nu_\mu + X \rightarrow l + X' + \pi$
Deep inelastic scattering	$\nu_\mu + N \rightarrow l + N' + m\pi(\eta, K)$

(*l* : lepton, N,N' : nucleon, m : integer)





3. Procedure of the neutrino event generation
(Event generator)

- 反応エネルギーの決定

  最終的な事象のエネルギー分布が
  neutrino flux (ϕ**(E)**) と total cross-section (σ_{total} **(E)**)
  の積の分布を再現するように。

- 反応モードの決定

  各反応モード毎の散乱断面積を用いて、反応種別を決定。
  （各反応の散乱断面積が計算できていることが必要）

- 素過程ニュートリノ反応のシミュレーション

  生成粒子数を決定、
  各粒子の粒子種別及び運動量（方向）を決定。

- 生成粒子の原子核中での散乱シミュレーション（必要なら）

  原子核中で発生した各粒子（ハドロン）の核内での散乱を
  シミュレーションし、各粒子が原子核から出た時の状態を決定。
  （吸収・散乱・粒子生成・荷電変換反応などがおこりうる）

3. Procedure of the neutrino event generation

1) Fix the energy of neutrino

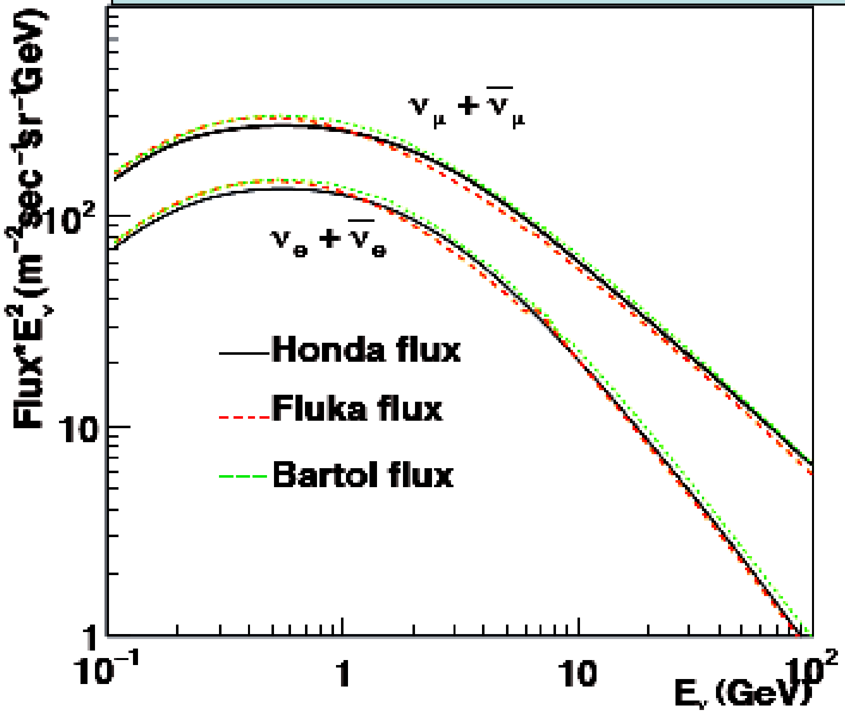
Energy distribution of neutrino should obey
the distribution determined by
neutrino flux (ϕ **(E)**) x total cross-section (σ_{total} **(E)**)

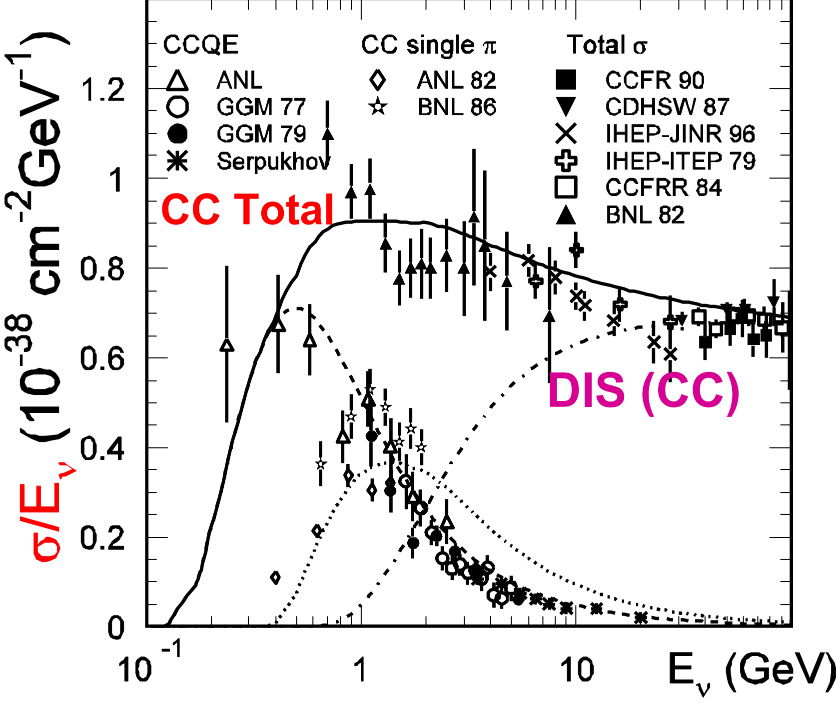
example)

Atmospheric neutrino

(Angular distribution is also taken into account in the actual simulation.)

neutrino flux (ϕ (E)) x Cross-sections





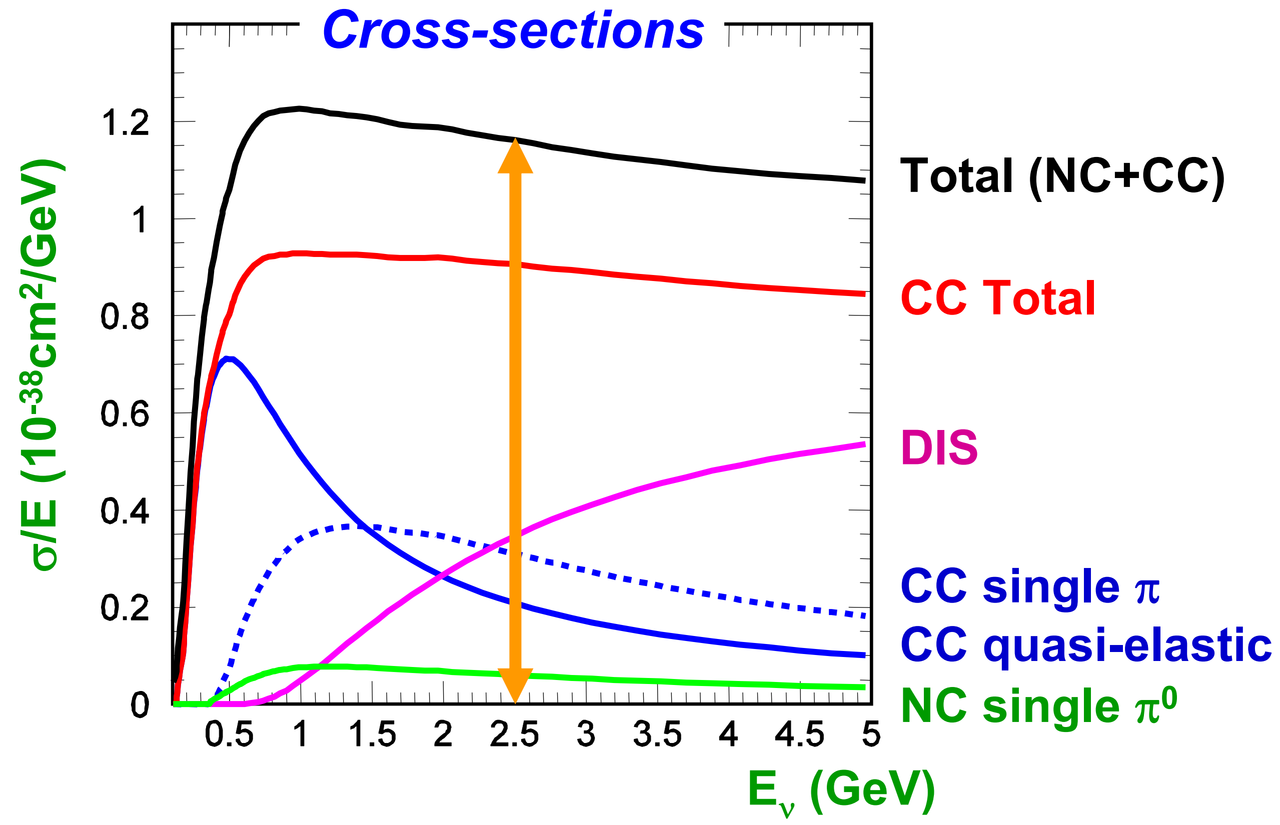
3. Procedure of the neutrino event generation

2) Fix the interaction mode

Interaction mode is selected by using the each individual interaction cross-section.

（In order to select one of the interactions it is necessary to know the cross-sections for each mode.）

Example) picked up energy in the previous step was 2.5GeV



3. Procedure of the neutrino event generation

3) Simulate primary interaction

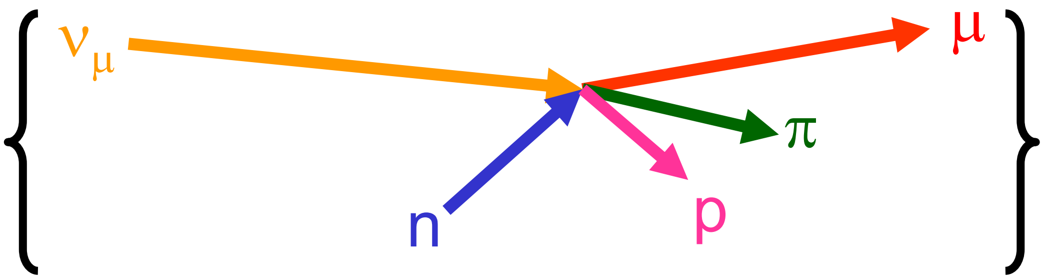
Fix the number of particles in the finals state

Fix the types and 4-momenta (direction) of each particle.

Example) single π production

Input : Use fixed neutrino energy and direction in step 1).

initial particles



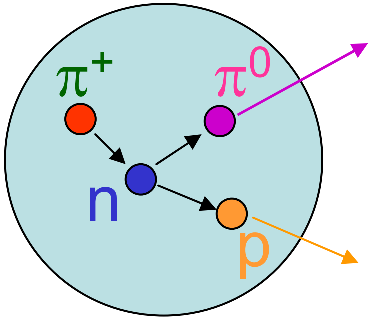
final particles

4) Simulate secondary interaction in nucleus

Simulate the interaction in the target nucleus and fix the properties of each particle outside of the target nucleus.

Example) π charge exchange interaction in nucleus

Trace generated hadrons in nucleus until the particles exit from the nucleus.



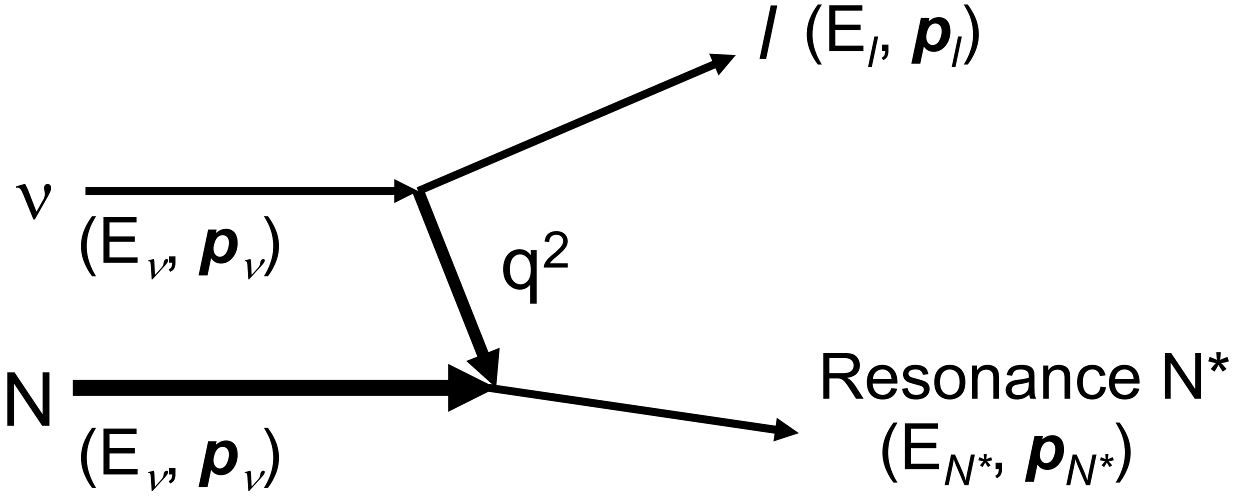
4. Introduction of NEUT ~ What NEUT does

- Neutrino energy range : ~ 100MeV to ~ TeV
- Target nucleus :primarily proton and Oxygen.
  Rather well tested with Carbon (at K2K, SciBooNE)

*) Possible to specify heavier or lighter target nucleus.
But additional approximations are used to simulate.

- Provide cross-sections to estimate the interaction rates
  or to select the interaction mode.
- Simulates primary neutrino interaction
  with nucleon and nucleus targets.
  Simulate emission of gamma from the excited state.
  (only for Oxygen at this moment)
- Simulates meson interactions in the target.
  Especially in detail for the low momentum pions.
- Simulates nucleon re-scattering in the target nucleus.

Before going to detail



l $(E_l, \boldsymbol{p}_l)$

ν $(E_\nu, \boldsymbol{p}_\nu)$

q^2

N $(E_\nu, \boldsymbol{p}_\nu)$

Resonance N*
$(E_{N^*}, \boldsymbol{p}_{N^*})$

q^2 : 4 momentum transfer

$$q^2 \equiv (E_l - E_\nu)^2 - (\boldsymbol{p}_l - \boldsymbol{p}_\nu)^2 \quad (= -Q^2)$$

W : Invariant Mass of N*

$$W \equiv \sqrt{E_{N*}^2 - p_{N*}^2}$$

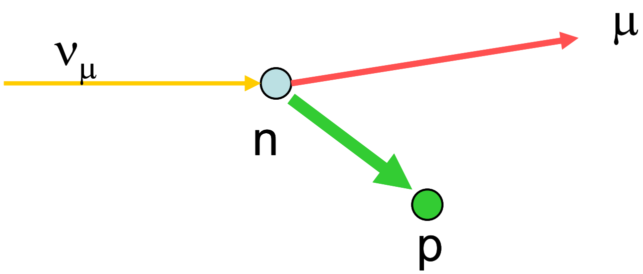
4. Introduction of NEUT

~ Summary of the interaction models

- QE
  - Llewellyn-Smith, Smith&Moniz
  - M_A is 1.1 or 1.2GeV/c
- Resonance production
  - Rein & Sehgal
  - M_A is 1.1 or 1.2GeV/c
  - Recently $\Delta \rightarrow N\gamma$ decay is considered.
  - Recently lepton mass effects in CC single-π is considered.
- Coherent-π
  - Rein & Sehgal

    (recently lepton mass correction is included.)
- DIS
  - GRV94 pdf with Bodek-Yang correction
  - GRV98 pdf with Bodek-Yang correction

Charged current quasi-elastic



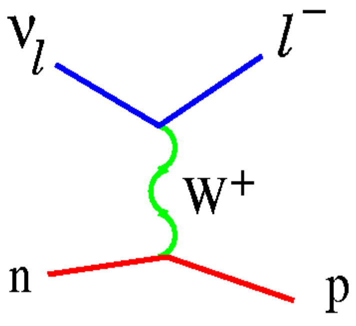
Major interaction in the low energy region

Useful interaction mode to reconstruct incident neutrino energy
~ used to measure the spectrum shape of the neutrino flux

Only the charged lepton is observed
if the momentum of recoil nucleon is below 1GeV
with the water Cherenkov detector.

5. Charged Current Quasi elastic scattering

ν + N → l + N’



Cross-section calculations

Free nucleon : C.H.L. Smith

(Phys. Rep. 3,261(1972))

$$\frac{d\sigma^\nu}{dQ^2} = \frac{m_N^2 G_F^2 \cos^2\theta_c}{8\pi E_\nu^2}\left[A(Q^2) \mp B(Q^2)\frac{(s-u)}{m_N^2} + C(Q^2)\frac{(s-u)^2}{m_N^4}\right]$$

$$G_E^V(Q^2) = \frac{1}{\left(1+Q^2/M_V^2\right)^2}, \quad G_M^V(Q^2) = \frac{1+\xi}{\left(1+Q^2/M_V^2\right)^2} \qquad \xi \equiv \mu_p - \mu_n = 3.71$$

$$F_A(Q^2) = \frac{-1.267}{\left(1+Q^2/M_A^2\right)^2}$$

(M_V=0.84GeV/c)

In the original article, both vector and axial-vector form factors are assumed to be dipole. Also, G_E^n is set to 0.

Recent precise electron scattering experiments suggests deviation from the dipole assumption for vector form factor.

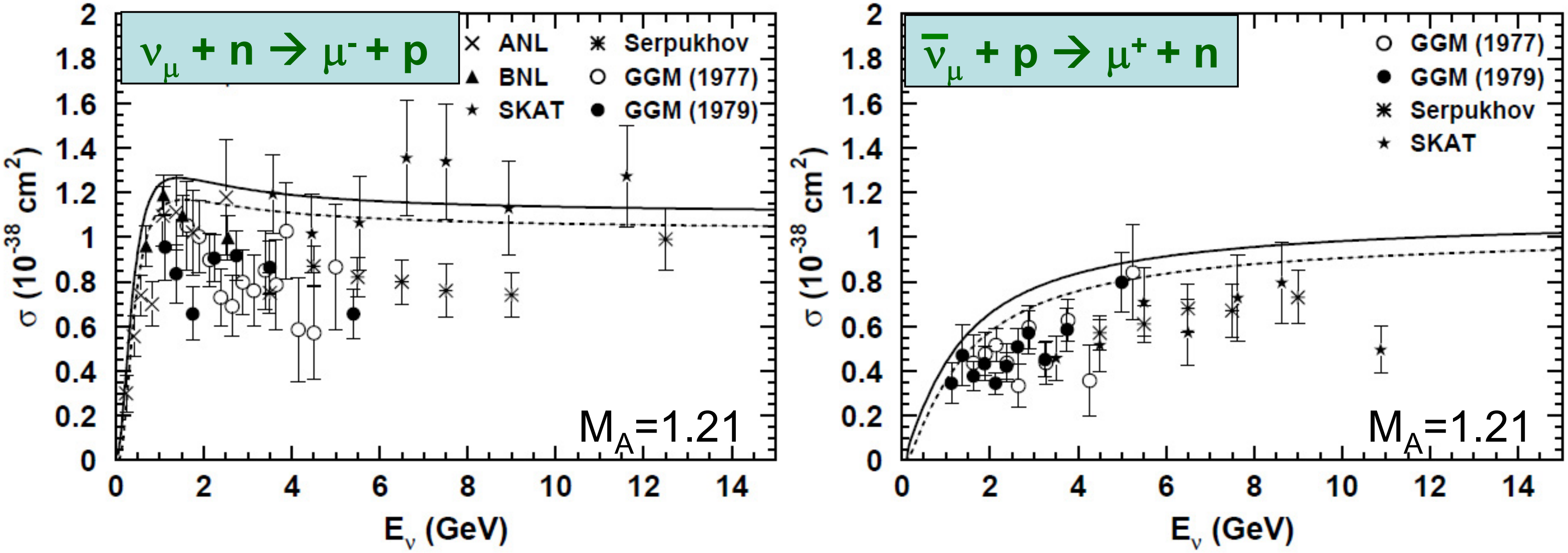
Still, dipole form factor is used in NEUT.

M_A = 1.1 or 1.2 GeV/c^2

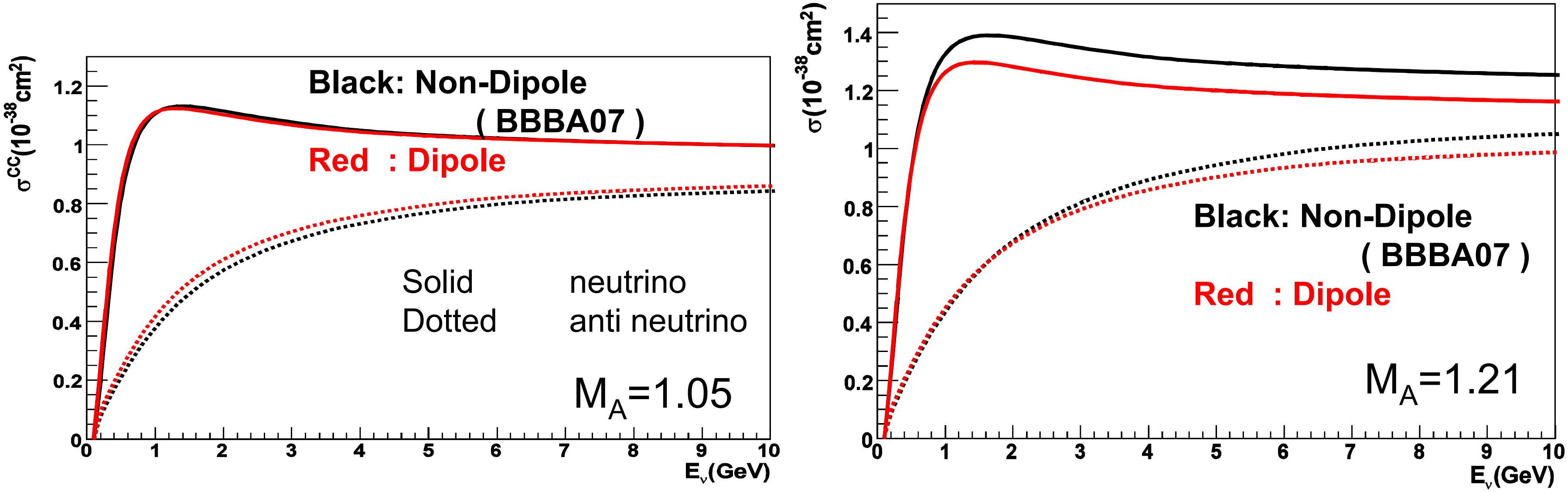
5. Charged Current Quasi elastic scattering

$\nu + N \rightarrow l + N'$

Cross-sections (Neut)



Cross-section differences (dipole and non-dipole vector form factors)



5. Charged Current Quasi elastic scattering

Quasi-elastic scattering with nucleon in nucleus

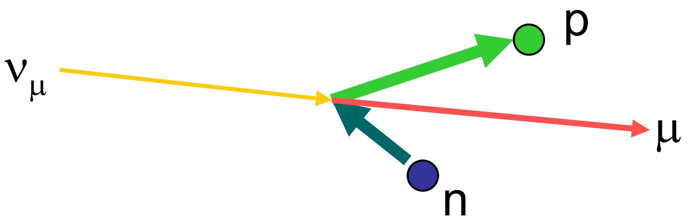
Nucleons are bound in nucleus.

$E_\nu > 100\text{MeV}$

→ Fermi gas model is a good (first-order) approximation.

- Nucleon is moving in the Fermi sea.
- Nucleon is bound in the nucleus. (“binding energy”)
- Pauli blocking is required.

(Outgoing nucleon required to have larger momentum than Fermi surface momentum.)



$$\begin{cases} E_\nu + E_n + B = E_p + E_\mu \\ p_p > \text{Fermi momentum} \end{cases}$$

To obtain the cross-section of CCQE for bound nucleons, relativistic Fermi-gas calculation by Smith and Moniz is used. (Nucl.Phys.B43 605(1972),erratum-ibid.B101 547(1975), with additional correction.)

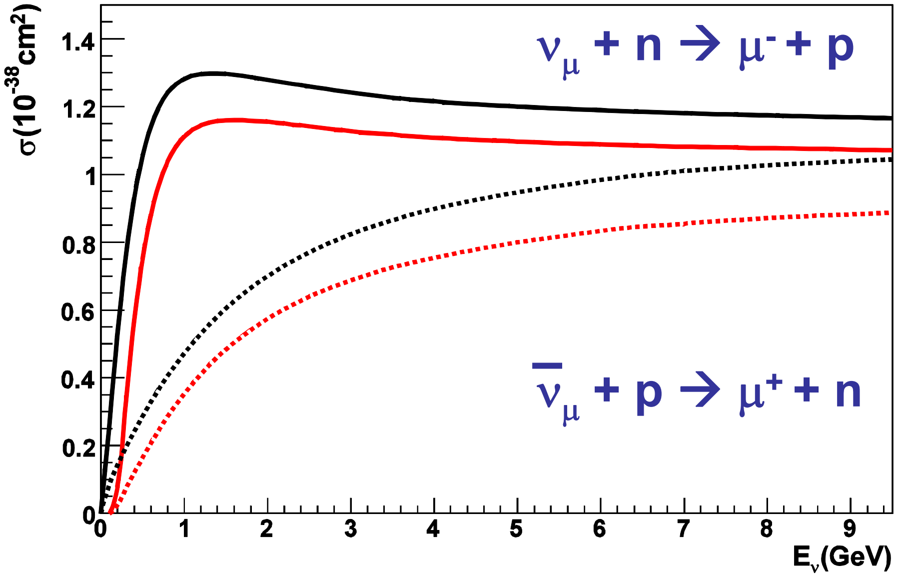
5. Charged Current Quasi elastic scattering

Assuming the simplest Fermi gas model,

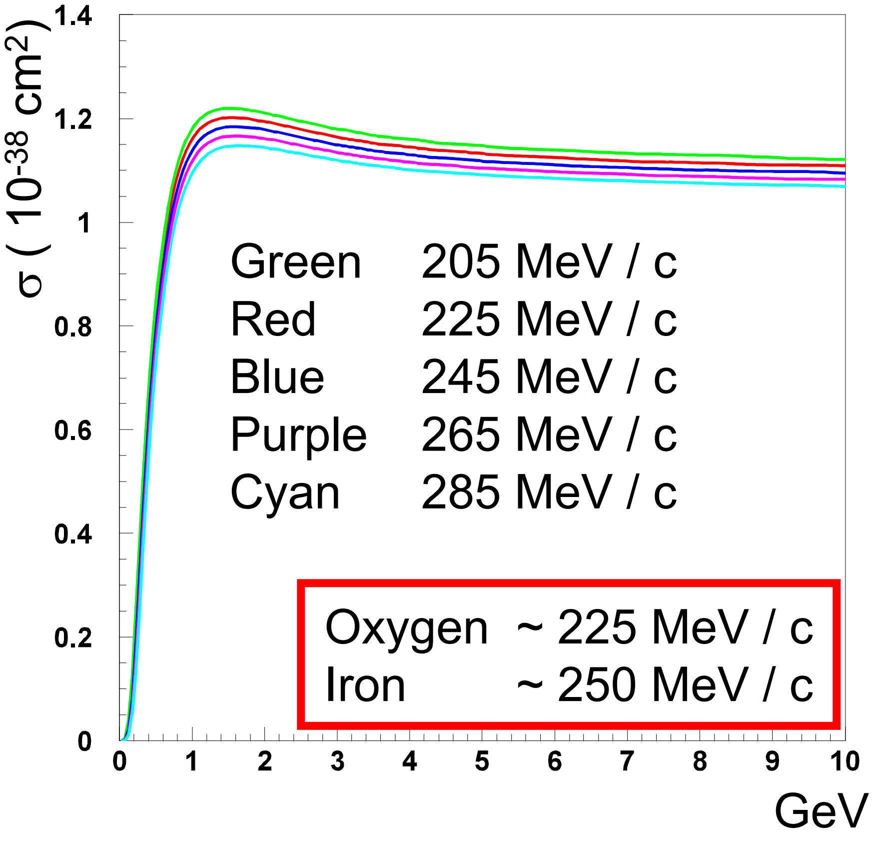
Target nucleus is determined by Fermi momentum and potential

Cross-section difference
(Free vs. bound)

Black	Free
Red	Bound



Cross-section difference
Fermi momentum dependence
~ A dependence



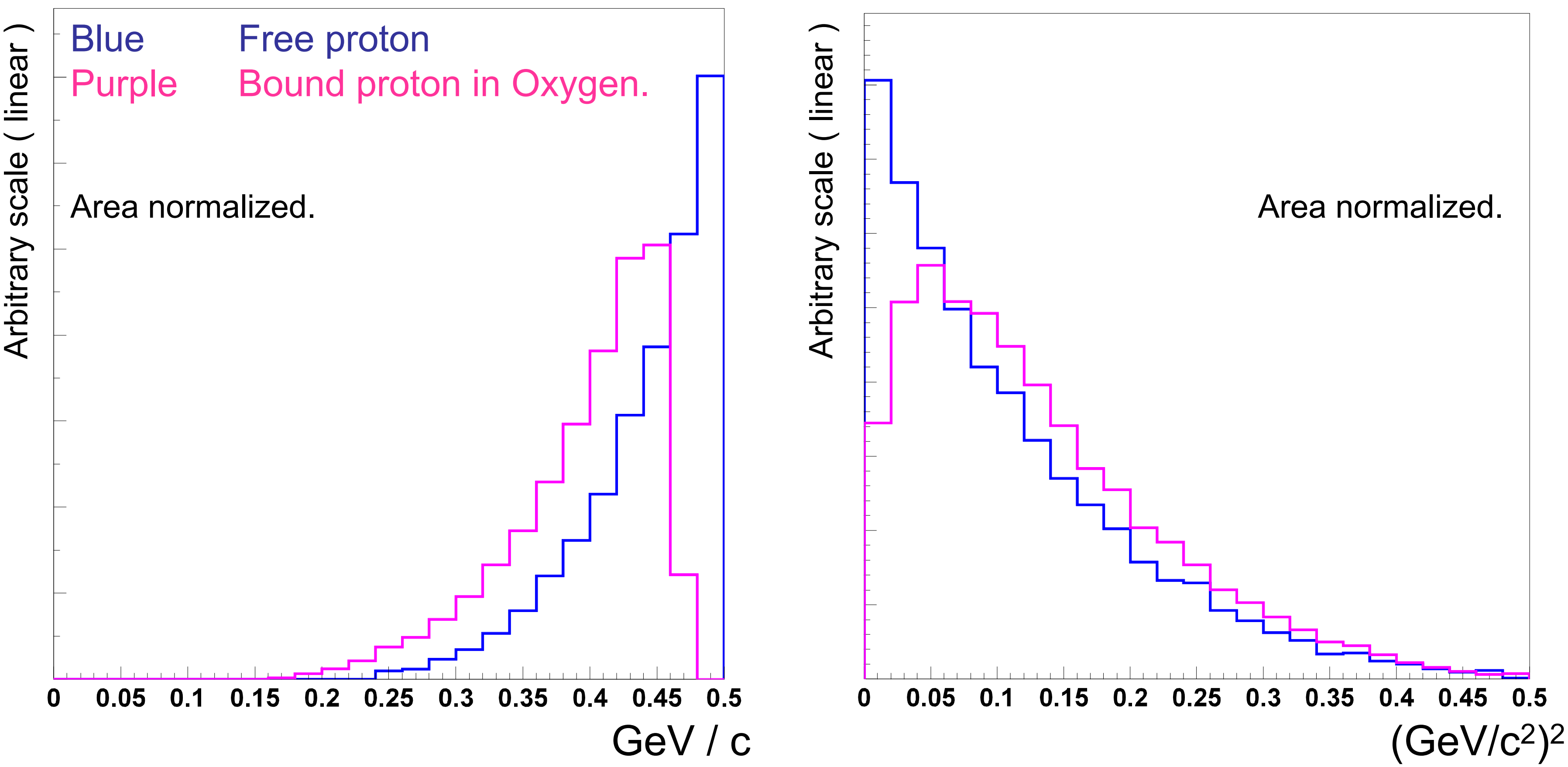
Oxygen	~ 225 MeV / c
Iron	~ 250 MeV / c

5. Charged Current Quasi elastic scattering

$\bar{\nu}_e + p \rightarrow e^+ + n$

Outgoing lepton momentum

q^2 (momentum transfer)



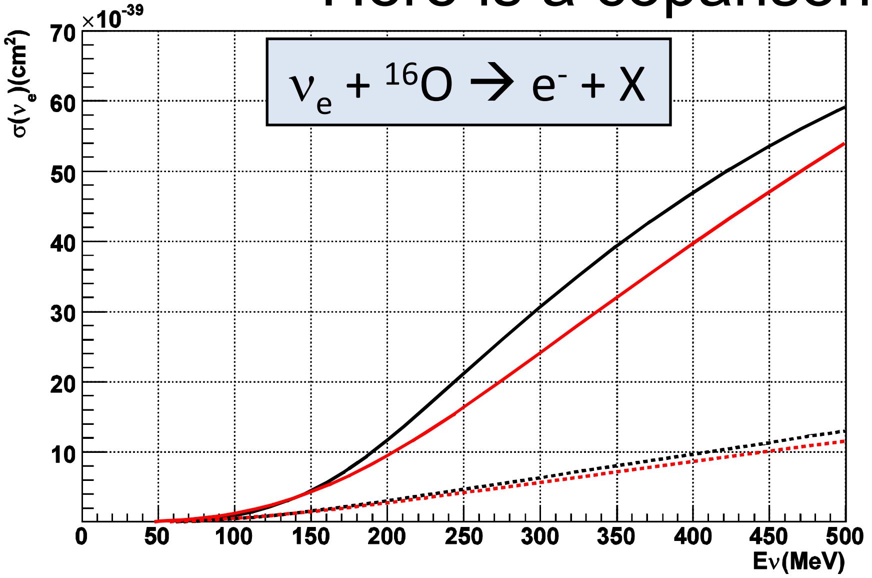
Clearly the small momentum transfer region
is suppressed due to Pauli Blocking and binding energy.

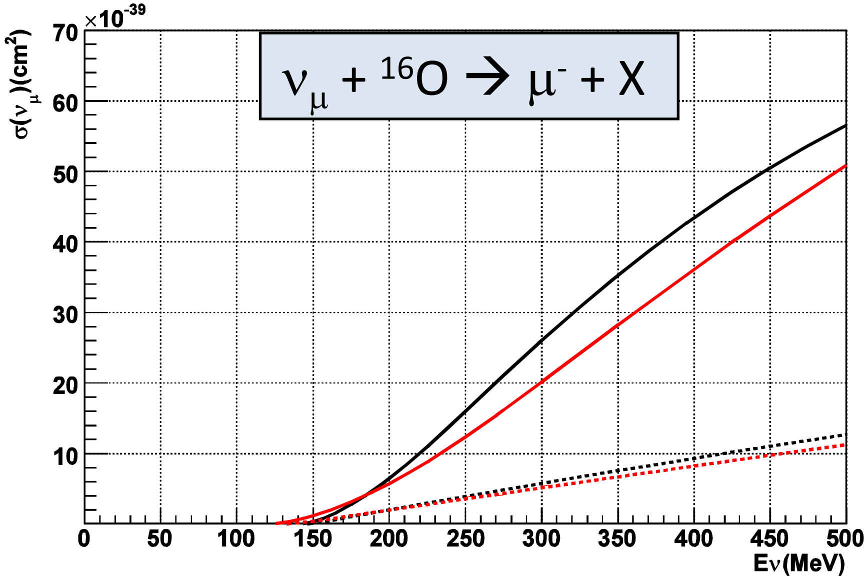
In the determination of the kinematics,
NEUT uses the classical (simple) energy and momentum conservations.

5. Charged Current Quasi elastic scattering

There are several recent models

Here is a coparison with the model by Nieves et al.





NEUT(Smith-Moniz)

Nieves et al.

solid: ν, dashed: $\bar{\nu}$

Nieves model is lower than S-M by 10% at 500MeV for both in ν_e,ν_μ

Ratio (Nieves to S-M) is within +/-25% above 200MeV.

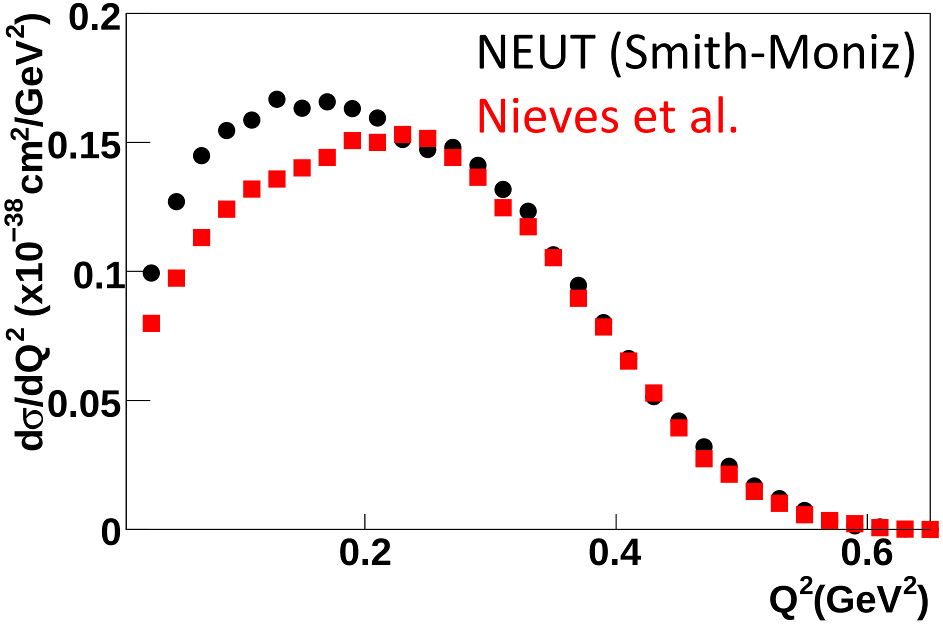
Currently, this model is applicable up to 500MeV, and not used in neut.

Authors are trying to extend the energy range up to ~1GeV.

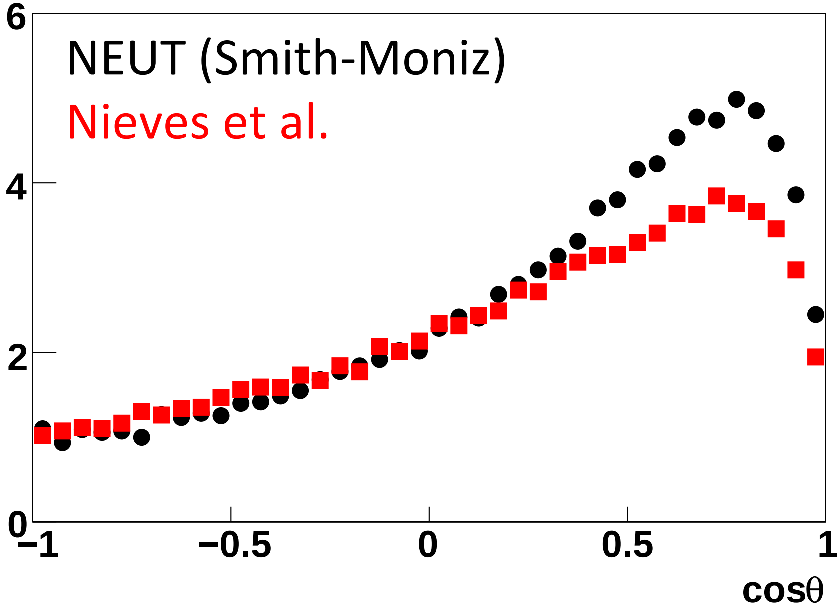
5. Charged Current Quasi elastic scattering

Comparison between the models
Smith-Moniz and Nieves et al.

$\nu_\mu + n \rightarrow \mu^- + p$



Differential cross-section dσ/dq²

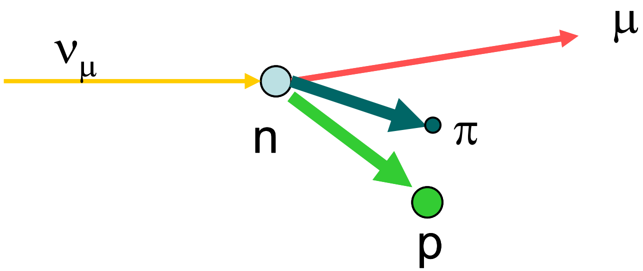


Angular distribution of μ

More detailed model by Nieves et al. shows
larger suppression in small q^2 region.

→ Forward going particles are suppressed.

Single pion production



Major background of the nucleon decay

Particles in the final state are same as the ones from nucleon decay.

Major background of the ν_e appearance search at T2K

γ from asymmetric decay of π^0
might be misidentified as ν_e appearance signal.

Major contamination to the energy spectrum measurement.

If pions are absorbed,
event shape is similar to the CCQE
in the water Cherenkov detector.

Without the detection of the recoil-nucleon track,
it is difficult to discriminate single pion and CCQE.

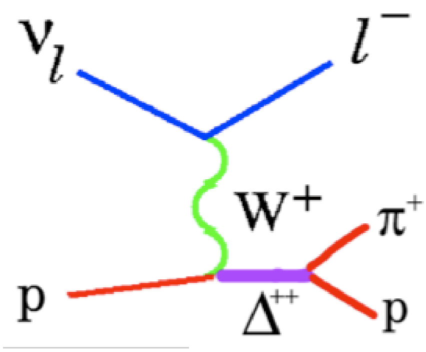
6. Single meson production via resonances

$\nu + N \rightarrow l + N' + \pi\ (K, \eta)$

Based on D.Rein, and L.M.Sehgal, Ann. of Phys. 133(1981)

$\nu + N \rightarrow l + \Delta\ (N^*)$

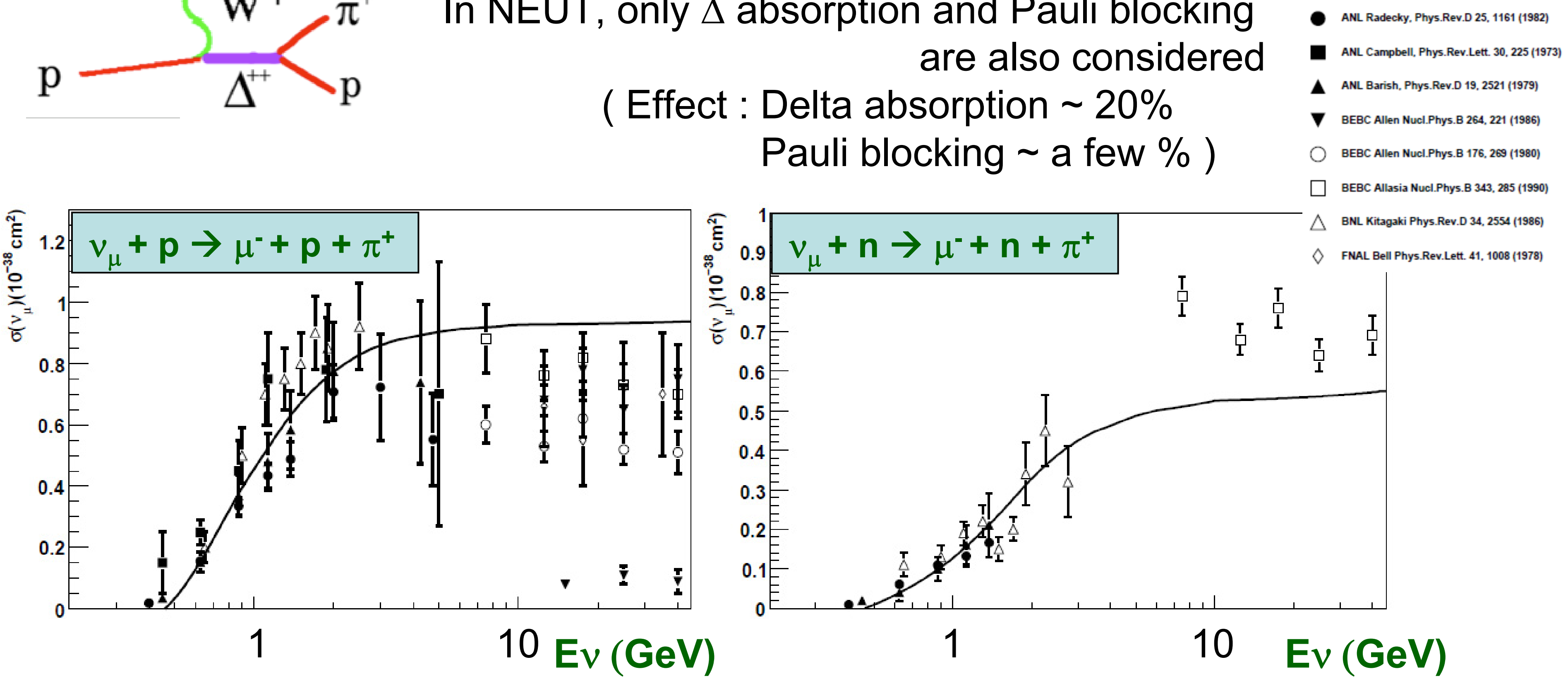
$\hookrightarrow \Delta\ (N^*) \rightarrow \pi + N'$



Resonances up to 2GeV are taken into account.

In NEUT, only Δ absorption and Pauli blocking are also considered

(Effect : Delta absorption ~ 20%
Pauli blocking ~ a few %)

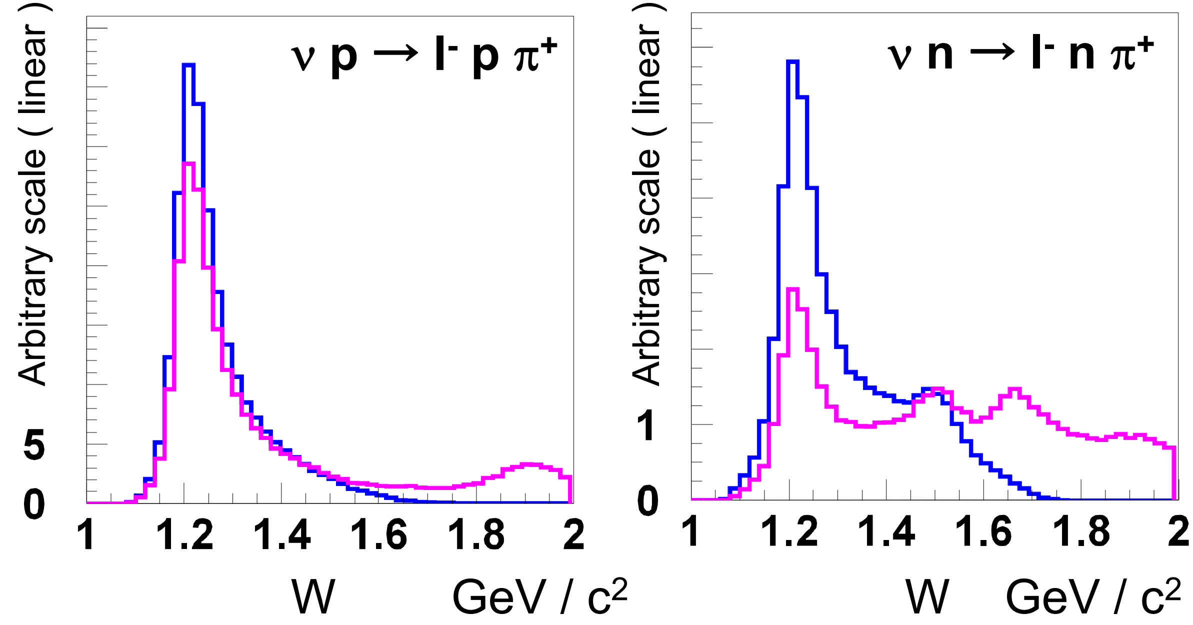


6. Single meson production via resonances

Invariant mass
of resonance (W)

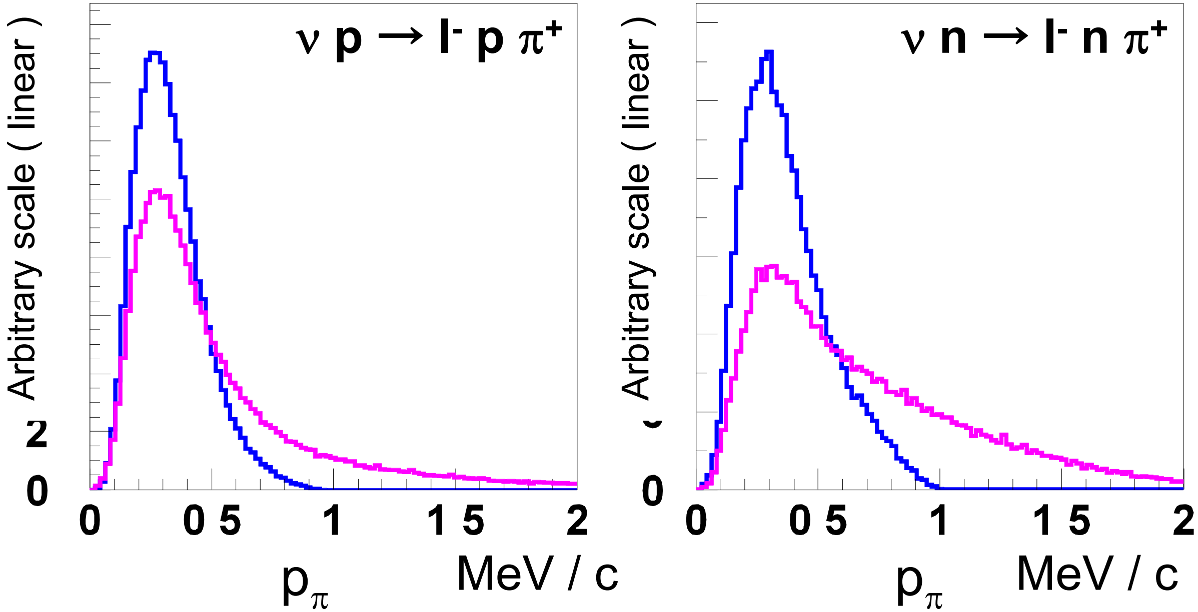
$\nu p \rightarrow l^- p \pi^+$

Δ^{++} (1232) dominant



momentum of π

first order :
determined by resonance masses



7. Single meson production via resonances

K and the other meson productions via resonances

Used to estimate the background of nucleon decay.

Basically same for the π production.

$\nu + N \rightarrow l + N^*$

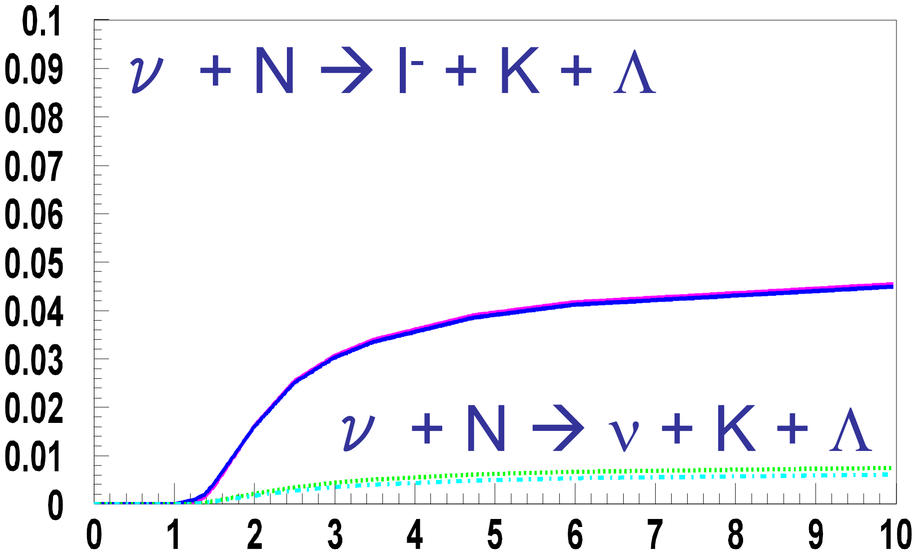
$\rightarrow \Delta(N^*) \rightarrow K + \Lambda$

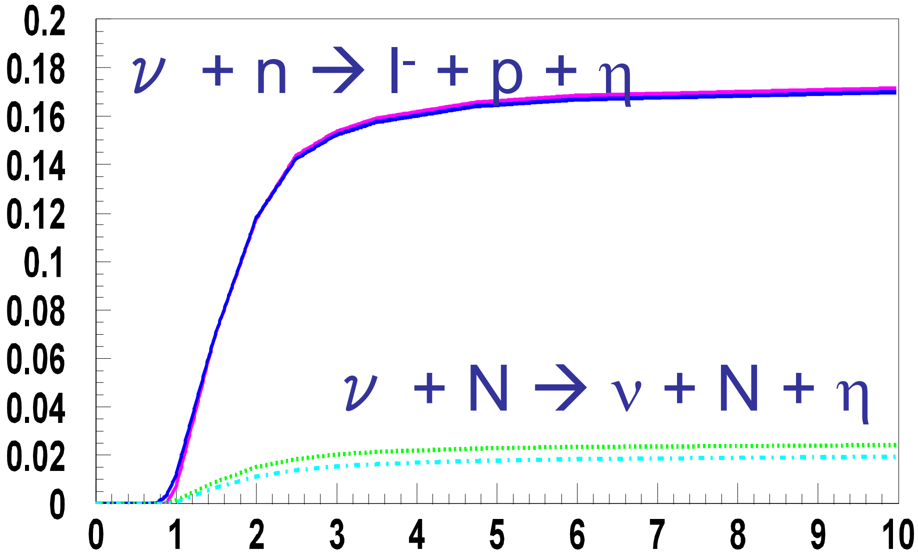
etc.

Use appropriate resonances and branching ratios.

In NEUT, we use upper limit of each branching ratio.

(for the background estimation)





8. $\Delta \rightarrow N\gamma$ decay

Δ resonance can decay into gamma (Br ~<0.5%)

This branching ratio is small,

but NC $\Delta \rightarrow N\gamma$ can be background

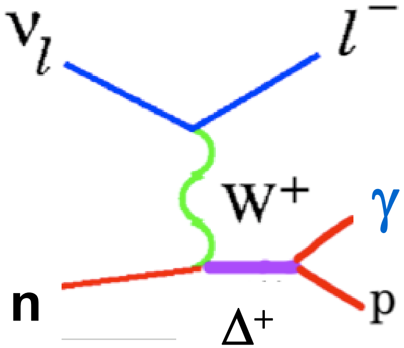
in searching for the $\nu_\mu \rightarrow \nu_e$ appearance.

This mode is added to NEUT, assuming

- Kinematics are basically same as $\Delta \rightarrow N\pi$

  $\nu + N \rightarrow l + \Delta\,(N^*)$

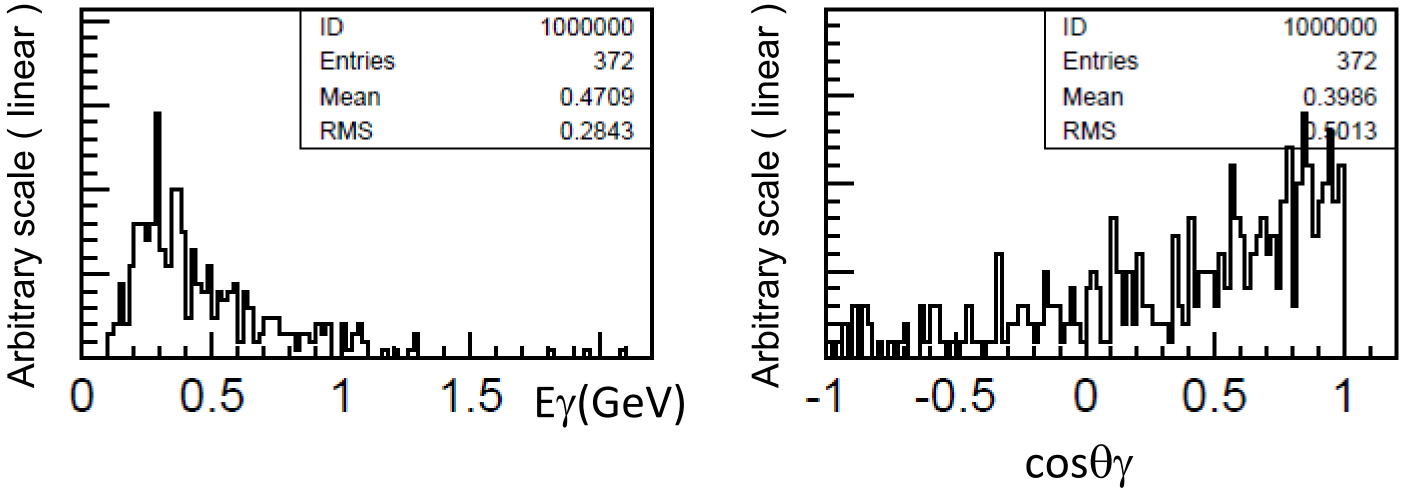
  $\quad\hookrightarrow \Delta\,(N^*) \rightarrow \gamma + N'$

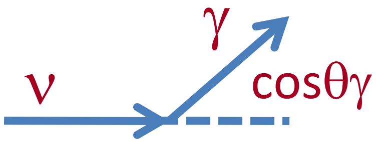


- Three reactions are considered
  - CC : $\nu + n \rightarrow l^- + p + \gamma$
  - NC : $\nu + n \rightarrow \nu + n + \gamma$
  - NC : $\nu + p \rightarrow \nu + p + \gamma$

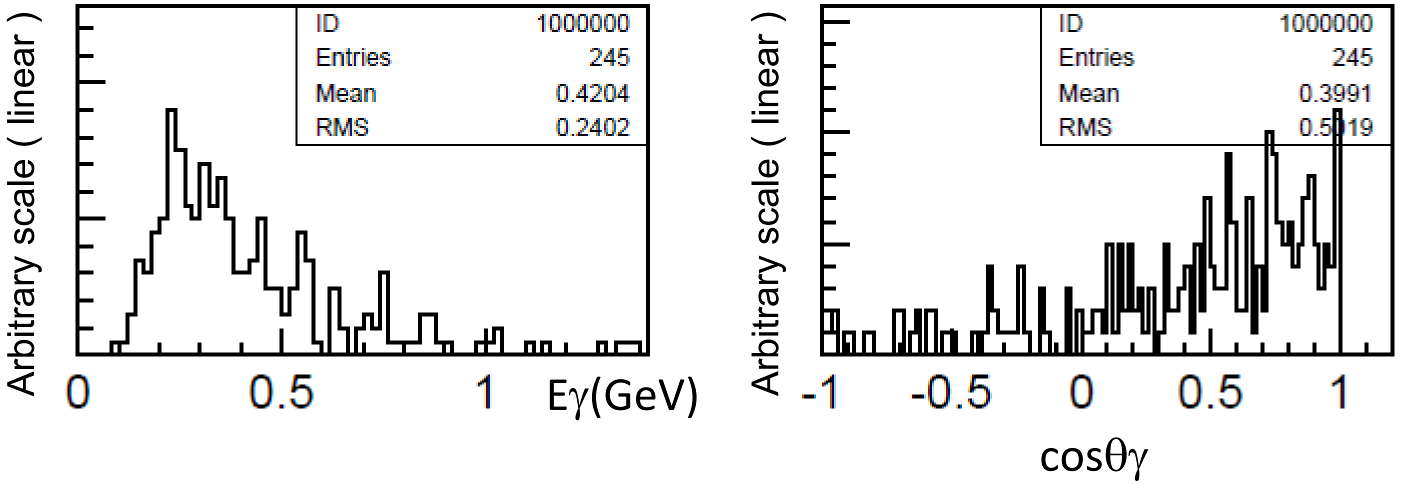
8. Δ → Nγ decay

CC : ν + n → l^- + p + γ





NC : ν + n → ν + n + γ



E_ν is randomly selected between 200MeV and 3000MeV

Deep inelastic scattering

Dominant interaction in the high energy region (> several GeV)

Understood as neutrino quark interaction.

Parton distribution function of nucleon is obtained
from the high energy lepton nucleon (nucleus) scattering experiments.

Thus, important in estimating the energy of neutrino
above several GeV.

(measure momentum of lepton
together with the total energy deposit
to estimate the energy of neutrino.)

9. Deep Inelastic scattering

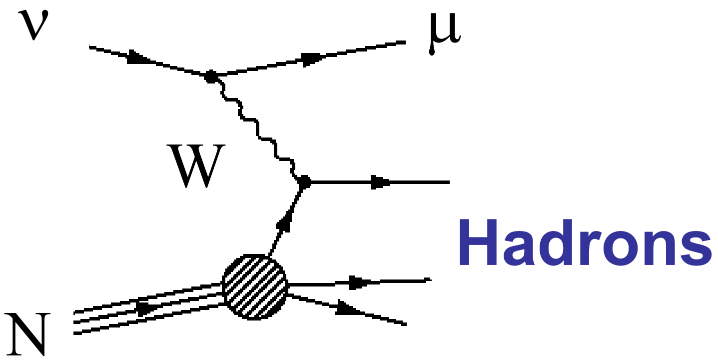
ν + N → l + hadrons

Dominant interaction in the high energy region (> several GeV)

$$\frac{d^2\sigma^\nu}{dxdy} = \frac{G_F^2 m_N E_\nu}{\pi}\left[(1-y+\frac{1}{2}y^2+C_1)F_2(x)+y(1-\frac{1}{2}y+C_2)[xF_3(x)]\right]$$

$$C_1 = \frac{m_\ell^2(y-2)}{4m_N E_\nu x}-\frac{m_N xy}{2E_\nu}-\frac{m_\ell^2}{4E_\nu^2},$$

$$C_2 = -\frac{m_\ell^2}{4m_N E_\nu x},$$



Parton distribution functions (F2 and xF3)

are extracted from the accelerator experiments.

However, major parton distribution functions

can not be used not applicable in the small W or q2 region.

(If we use the PDF as-is,

excess was observed in the small q^2 region.)

1) Use experimental results of neutrino scattering in that region.
2) Apply corrections to the parton distribution functions.

9. Deep Inelastic scattering ν + N → l + hadrons

Avoid double counting : the resonance region to the DIS region

W < 2GeV : Restrict # of mesons to be larger than 1

Exclude 1 meson production

by using multiplicity function <n>(W)

Because non-resonant background is already included

in the single π production.

Multiplicity is determined based on the experimental result.

Current version: S. J. Barish et al. Phys. Rev D.17,1 (1978)

(There are recent reports from CHORUS collaboration.

Eur.Phys.J.C51:775-785,2007)

$$\langle n_\pi \rangle = 0.09 + 1.83 \ln(W^2)$$

W > 2GeV : Use PYTHIA to generate vectors.

	W < 2GeV	W > 2GeV
# of π = 1	Rein & Sehgal	PDF + Custom kinematics (Bodek & Yang Corr.)
# of π > 1	Use PDF + PYTHIA (Bodek & Yang Corr.)	Use PDF + PYTHIA (Bodek & Yang Corr.)

As for the parton distribution function,

we use the correction suggsted by Bodek and Yang.

9. Deep Inelastic scattering

Bodek and Yang (For GRV98)

1. Modified scaling variable

$$\xi_w = x\frac{Q^2+B}{0.5Q^2(1+[1+(2Mx)^2/Q^2]^{1/2})+Ax}$$

2. Correction factor for the PDF (K-factor) to describe low q^2

$$K_{valence} = \frac{[1-G_D^2(Q^2)][Q^2+C_{2v}]}{Q^2+C_{1v}}$$

$$1-G_D^2(Q^2) = \frac{Q^2}{Q^2+C}$$

$$K_{sea} = \frac{Q^2}{Q^2+C_{sea}}$$

A = 0.419

B = 0.223

C_{1v} = 0.544

C_{2v} = 0.431

C_{sea} = 0.380

3. Correction to Callan-Gross relation

$$2xF_1 = F_2\frac{1+4Mx^2/Q^2}{1+R}$$

R: Fitted function

4. d/u ratio

$$u_v \rightarrow u_v'(d_v,u_v) \quad d_v \rightarrow d_v'(d_v,u_v)$$

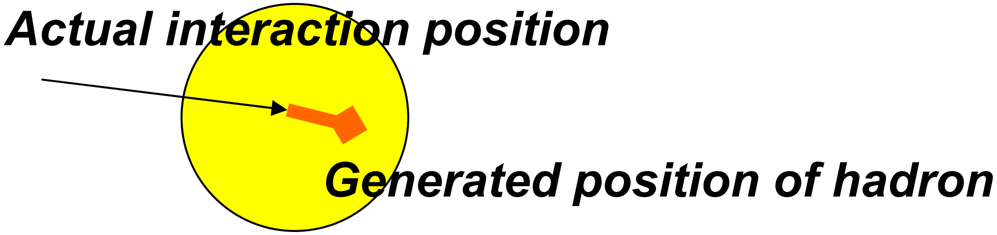
9. Formation zone

In determining the location of the interaction,
the simple Woods-Saxon density function is used.

In order to pick up the location of the production point
of hadrons,
Formation Zone is taken into account.

The idea of formation zone:

Hadronization does not occur at the interaction point
Distance from the neutrino interaction point
to the production point of hadron
is proportional to the momentum of hadron.



Formation Zone (L) is defined as follows:

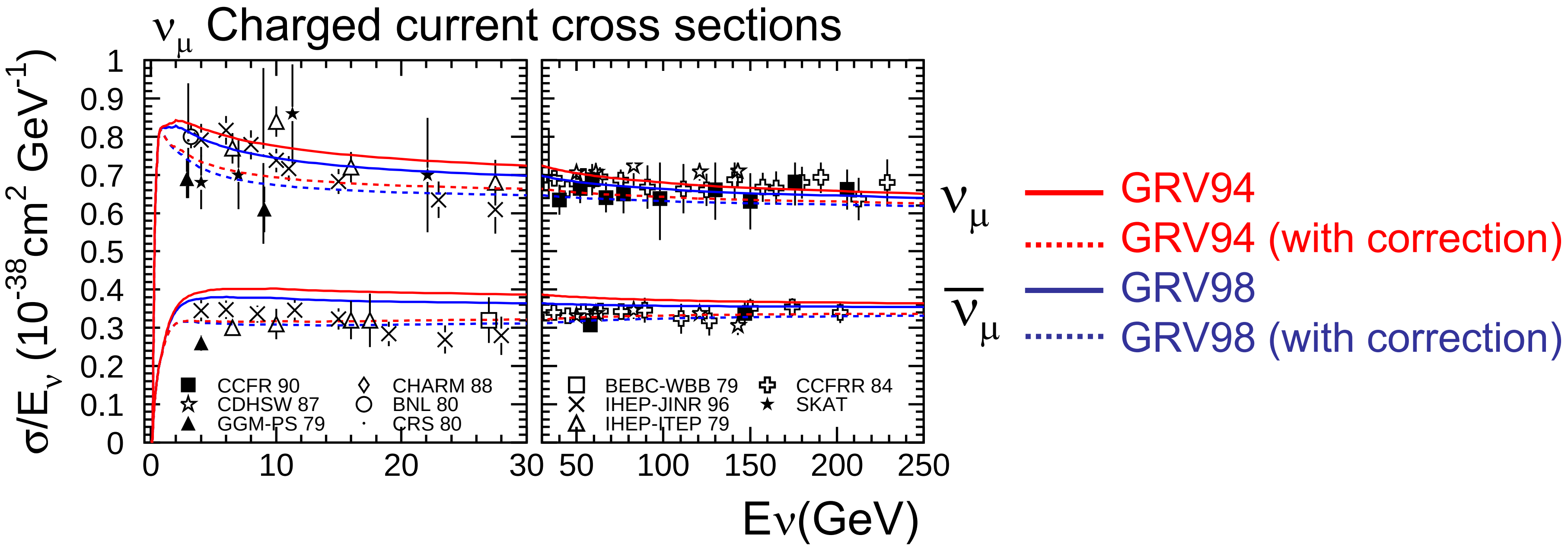
$$L = p / \mu^2$$

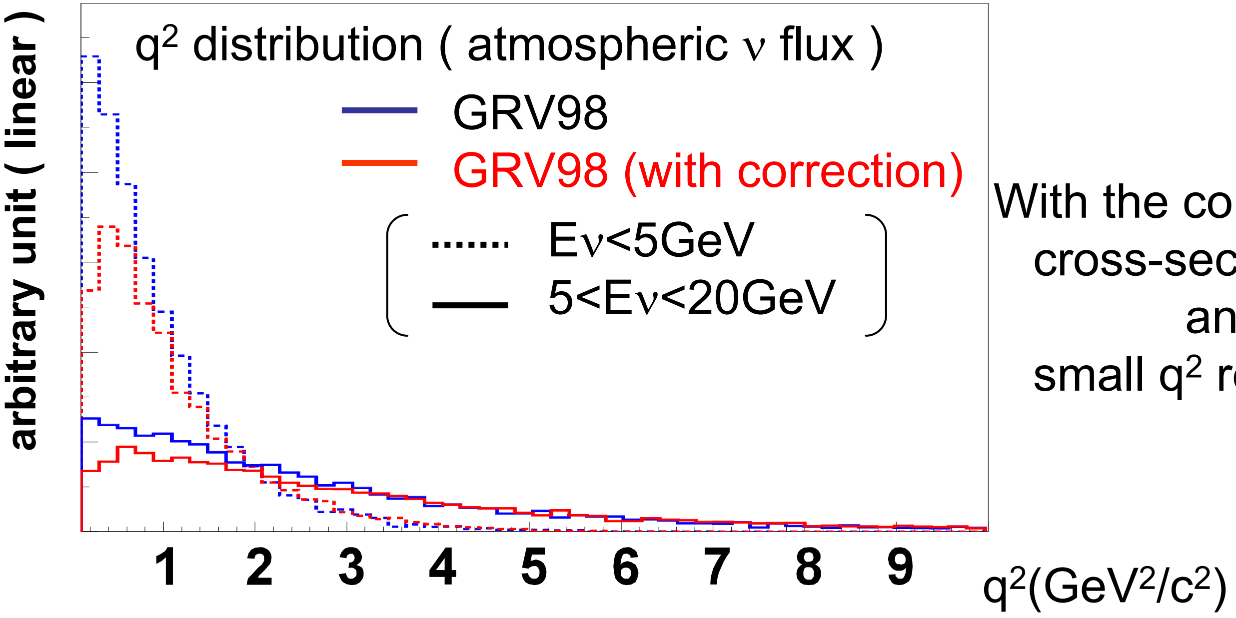
p : Momentum of hadron
μ^2: fitted constant = $0.08 \pm 0.04\ \mathrm{GeV}^2$

Ammosov et al.

9. Deep Inelastic scattering

(G. Mitsuka)





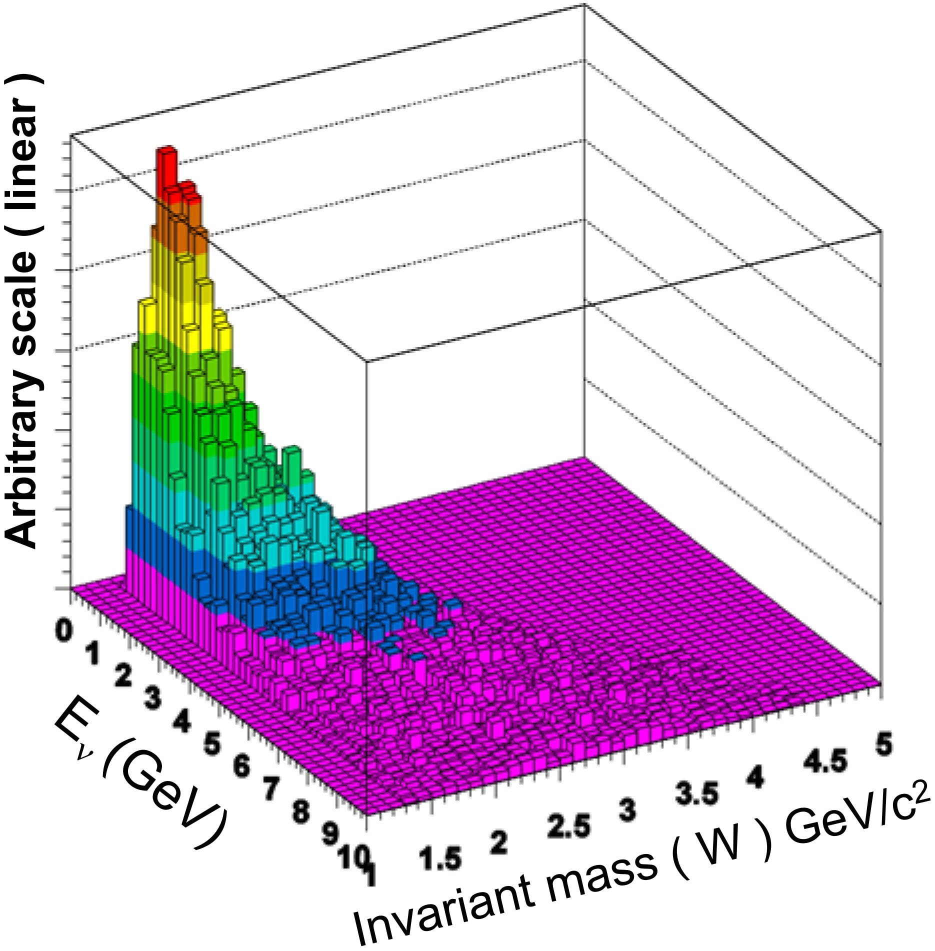
With the correction,
cross-section is suppressed
and
small q^2 region is also suppressed.

9. Deep Inelastic scattering

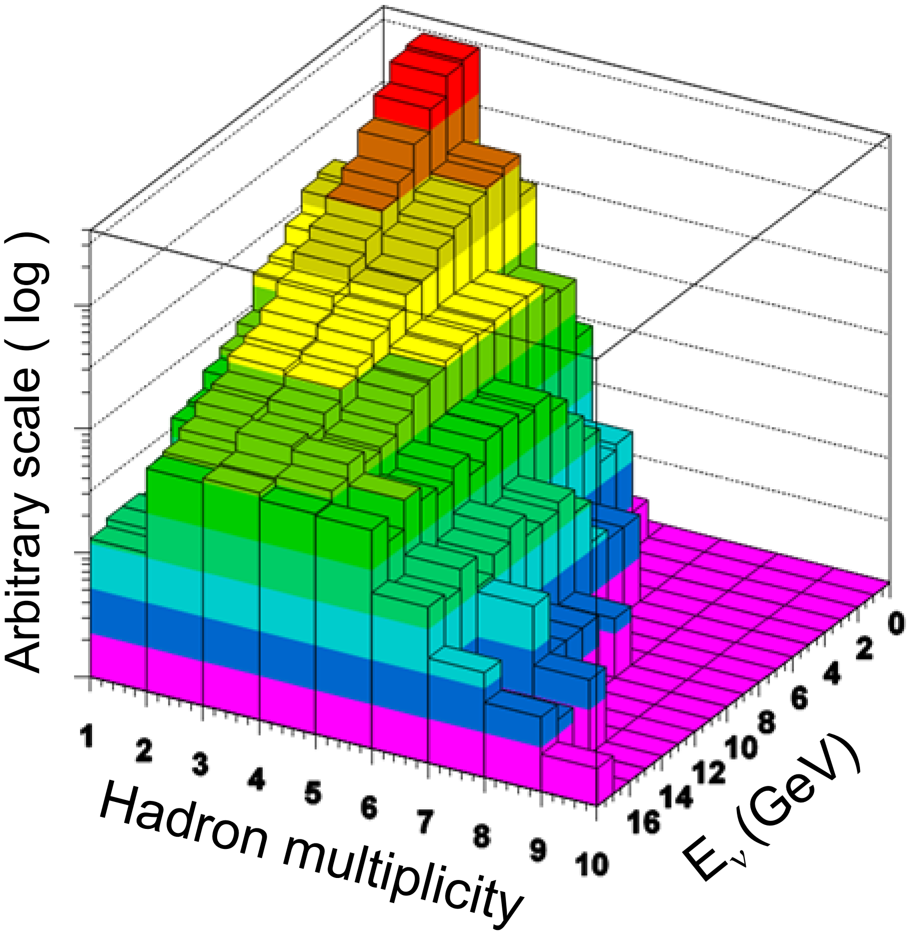
Two examples of basic distributions

(Atmospheric neutrino flux was used as input)

Invariant mass (W) vs. E_ν



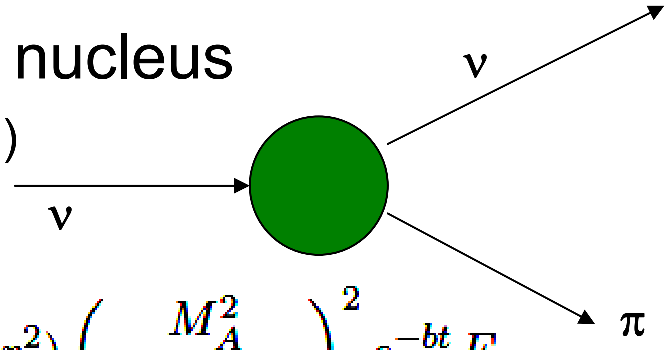
Hadron multiplicity vs. E_ν



10. Coherent pion productions $\overset{(-)}{\nu} + X \rightarrow \overset{(-)}{\nu} + X + \pi^0$

π production without breaking the target nucleus

Model by Rein & Sehgal (Nucl.Phys.B223:29,1983)

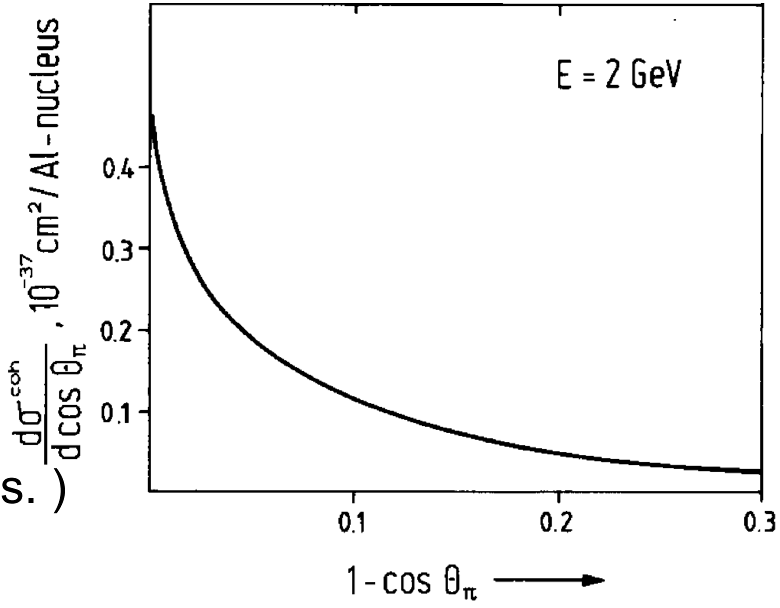


$$\frac{d^3\sigma}{dQ^2dydt} = \frac{G_F^2 m_N E_\nu}{2\pi^2} f_\pi^2 A^2(1-y)\frac{1}{16\pi}(\sigma_{\rm tot}^{\pi N})^2(1+r^2)\left(\frac{M_A^2}{M_A^2+Q^2}\right)^2 e^{-bt} F_{\rm abs} ,$$

$$r = \frac{\mathrm{Re}[f_{\pi N}(0)]}{\mathrm{Im}[f_{\pi N}(0)]} ,$$

- Cross-section is smaller than the resonance-mediated mode.
- Direction of π has peak in forward

(Experimentally observed
in the higher energy neutrino experiments.)



$\overset{(-)}{\nu} + {}^{12}C \rightarrow l^{\pm} + {}^{12}C + \pi^0$

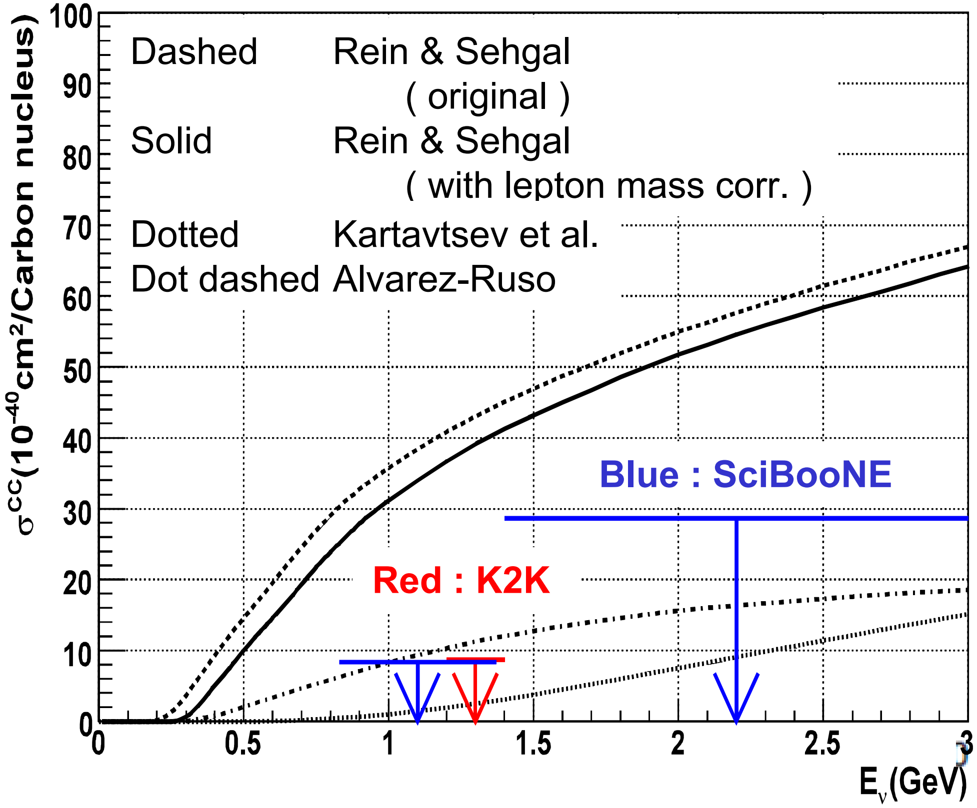
Recently, cross-section of **charged current** coherent pion production was found to be very small in ~<GeV region.

M. Hasegawa et al.(K2K collaboration) (hep-ex/0506008)

10. Coherent pion productions

Low energy charged current coherent pion production seems to be small.
(Results from the K2K and the SciBooNE experiments.)

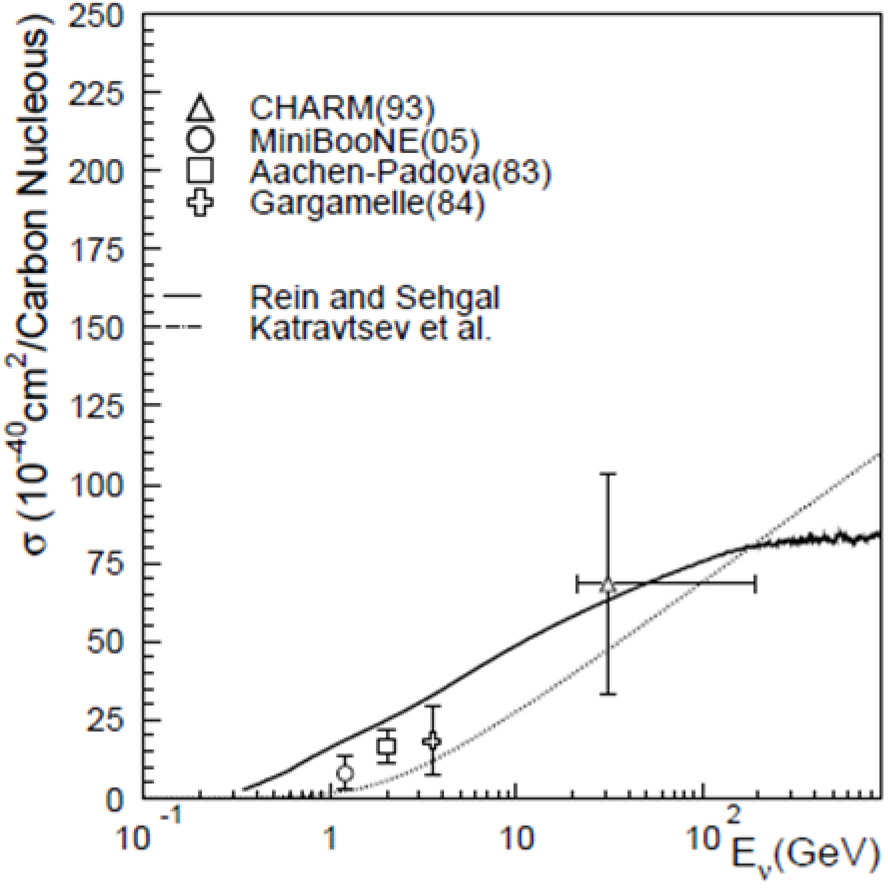
$\overset{(-)}{\nu} + {}^{12}C \rightarrow l^{\pm} + {}^{12}C + \pi^{\pm}$



Blue : SciBooNE

Red : K2K

$\overset{(-)}{\nu} + X \rightarrow \overset{(-)}{\nu} + X + \pi^0$

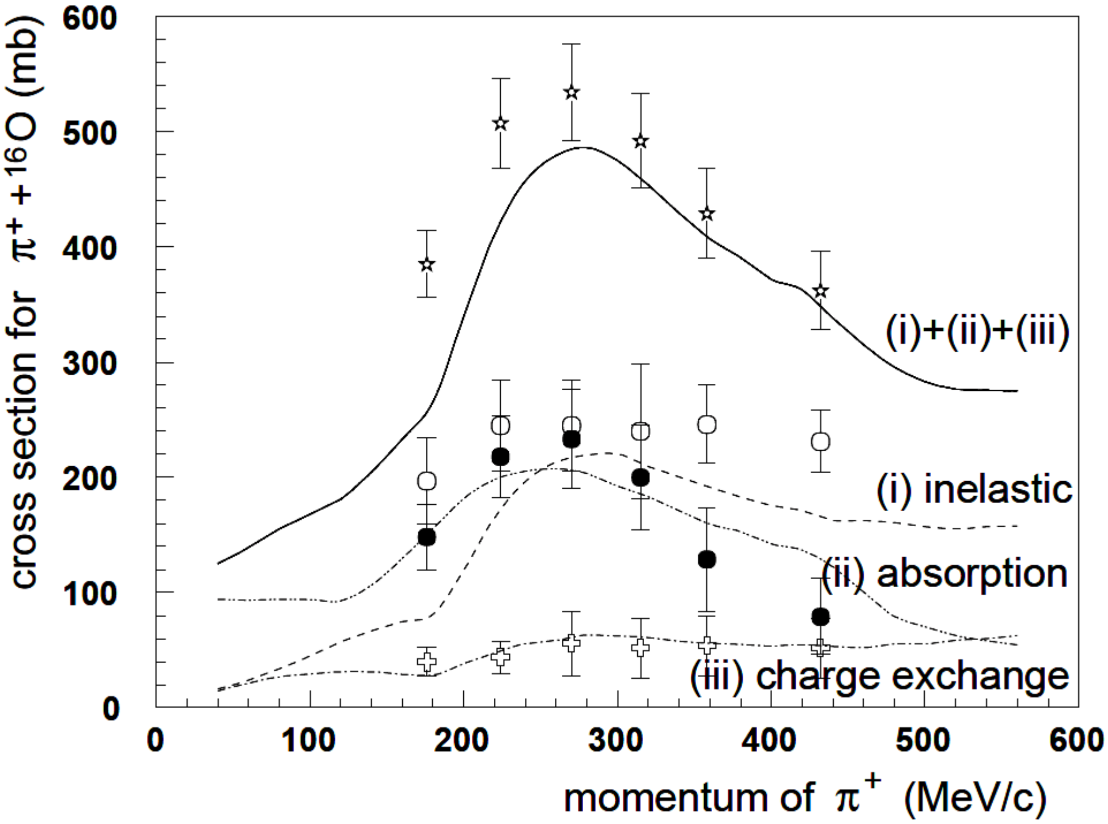


12. Nuclear effects (Final state interactions of hadrons)

Large fraction of the π from single π production are coming from the decay of Δ.

Cross-section of those π is large.

Interaction probability of π in nucleus generated by the single π production is large.



Momentum of π from $p \to e^+ \pi^0$ is ~460<MeV/c.

11. Nuclear effects (Final state interactions of hadrons)

re-scattering of pion, kaon, eta, omega and nucleon in nucleus

Different models are used in each simulation program.

Implementation in NEUT

Cascade model is used.

Each particle is tracked in the nucleus
until it escapes from the nucleus.

For low momentum pion (< 500MeV/c , so-called Δ region)

Mean free paths of absorption and inelastic-scattering
are calculated based on a model by L.Salcedo et al. .

(Nucl. Phys. A484(1998) 79)

* These mean free paths are position and momentum dependent.
* The Fermi surface momentum also has radius dependence.

For the higher momentum pion (> 500MeV/c),
kaons, eta, omega and nucleons

Parameters are taken from various experiments.

11. Nuclear effects (Final state interactions of hadrons)

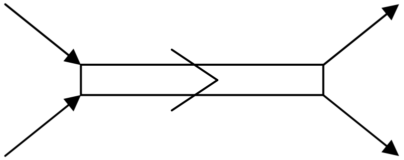
Low momentum pion (< 500MeV/c , so-called Δ region)

(L.L.Salcedo et al. Nucl. Phys. A484(1998) 79)

Calculate interaction probability using Δh model
and π & Δ self-energy.

Pion scattering

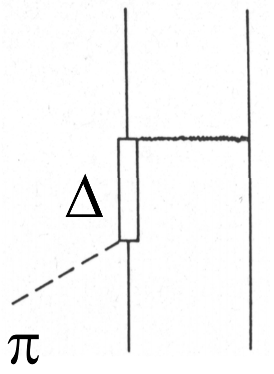
$$\pi + \mathbf{N} \rightarrow \Delta \rightarrow \pi + \mathbf{N}$$



Pion absorption

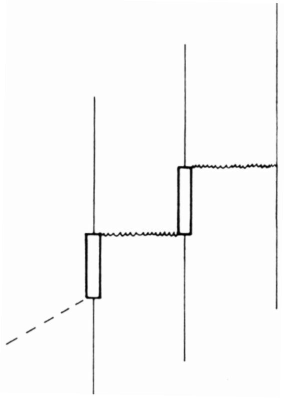
$$\pi + \mathbf{N} + \mathbf{N} \rightarrow \Delta + \mathbf{N} \rightarrow \mathbf{N} + \mathbf{N}$$ (2 body)

(2 body)



$$\pi + \mathbf{N} + \mathbf{N} \rightarrow \Delta + \mathbf{N} \rightarrow \mathbf{N} + \Delta \rightarrow \mathbf{N} + \mathbf{N}$$ (3 body)

(3 body)



(Note: There are several other diagrams. See reference.)

11. Nuclear effects (Final state interactions of hadrons)

$\pi^+ p \rightarrow \pi^+ p$ Interaction probability

$$P(\pi^+ p \to \pi^+ p) = \frac{1}{\omega} 2 \int \frac{d^3 k}{(2\pi)^3} n(\mathbf{k}) \frac{2}{3} \left(\frac{f^*}{m_\pi} \right)^2 \mathbf{q}^2{}_{c.m.} \left| G_\Delta (q+k) \right|^2 \frac{1}{2} \widetilde{\Gamma}(q+k)$$

Δ propagater

$$G_\Delta = \left(\sqrt{S} - M_\Delta + \frac{1}{2} i\Gamma \right)^{-1}$$

Corrected width of Δ

$$\frac{1}{2} \widetilde{\Gamma}(q+k) = \frac{1}{2} \Gamma \times \frac{1}{4} (\overline{\mu}^3{}_{c.m.} + \overline{\mu}_{c.m.} + 2)$$

$$\overline{\mu}_{c.m} = \begin{cases} -1 & (\text{ if } \mu^0 < -1) \\ \mu^0 & (\text{ if } -1 \leq \mu^0 \leq 1) \\ 1 & (\text{ if } \mu^0 > 1) \end{cases}$$

Use local density to determine k_F when considering Pauli blocking effect.

$$\mu^0 = \frac{E_\Delta E^N{}_{c.m.} - E_F W}{|\mathbf{q} + \mathbf{k}| |\mathbf{q}_{c.m.}|}$$

$$E_\Delta = q^0 + k^0$$

$$E_{c.m.} = \sqrt{q^2{}_{c.m.} + M^2}$$

$$E_F = \sqrt{k_F{}^2 + M^2}$$

$$W = \sqrt{s}$$

11. Nuclear effects (Final state interactions of hadrons)

Absorption probability

$$P = \frac{1}{\omega}\frac{4}{9}\left(\frac{f^*}{m_\pi}\right)^2 \mathbf{q}^2{}_{c.m.}\left|\widetilde{G}_\Delta(\overline{q+k})\right|^2\left(\frac{1}{2}\widetilde{\Gamma} - \mathrm{Im}\Sigma_\Delta\right)\rho$$

$$\mathrm{Im}\Sigma_\Delta(\omega) = -[C_Q(\rho/\rho_0)^\alpha + C_{A2}(\rho/\rho_0)^\beta + C_{A3}(\rho/\rho_0)^\gamma]$$

$$C(T_\pi) = ax^2 + bx + c \qquad x = T_\pi/\mu$$

E.Oset et al. Nucl. Phys. A468 (1987) 631

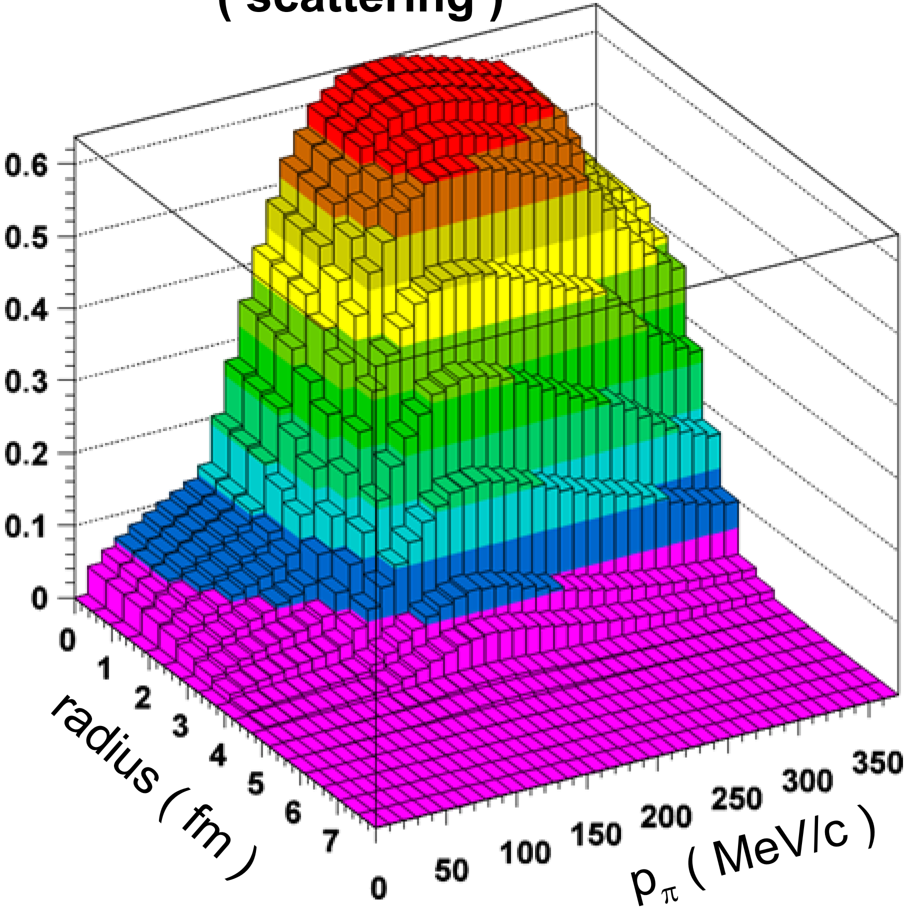
	C_Q (MeV)	C_{A2} (MeV)	C_{A3} (MeV)	α	β
a	-5.19	1.06	-13.46	0.382	-0.038
b	15.35	-6.64	46.17	-1.322	0.204
c	2.06	22.66	-20.34	1.466	0.613

$\gamma=2\beta$

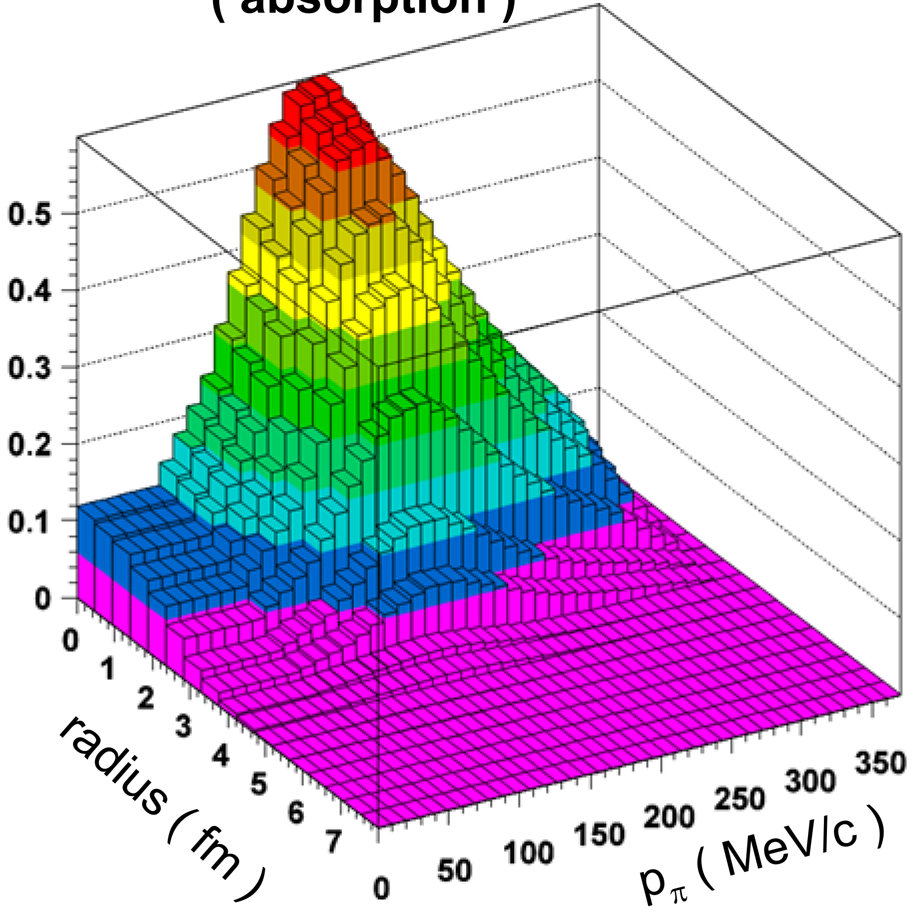
11. Nuclear effects (Final state interactions of hadrons)

Interaction probability (mean free path) depends on the π momentum and location (radius ~ density).

Interaction probability (scattering)



Interaction probability (absorption)



Fermi surface momentum depends on the local density.

Smaller q^2 interaction is allowed at the peripheral region

11. Nuclear effects (Final state interactions of hadrons)

Kinematics of the scattered particles

Use the results of phase shift analysis of π-N scattering

Also, the medium correction is applied to each phase shift.
(R.Seki et al., Phys. Rev. C27 (1983) 2817)

$$f=\sum_T C_T \sum_l \{[l f^l_{2T,2l-1}+(l+1)f^l_{2T,2l+1}] \times P_l(\cos\theta)-i\boldsymbol{\sigma}\cdot\boldsymbol{n}[f^l_{2T,2l-1}-f^l_{2T,2l+1}] \times P'_l(\cos\theta)\}\,.$$

Here, $f^l_{2T,2J}$ is an amplitude with orbital angular momentum l, isospin T, and total angular momentum J. C_T is the isospin factor written with Clebsch-Gordan coefficients,

$$C_T=\left(1t_{\pi'}\,\frac{1}{2}\,t_{N'}\,|\,Tt_\pi+t_N\right)\left(1t_\pi\,\frac{1}{2}\,t_N\,|\,Tt_\pi+t_N\right),$$

(t_N, t_π, $t_{N'}$, $t_{\pi'}$ are initial and final Z component of π, N isospin.)
$\boldsymbol{\sigma}$ is the Pauli matrix and $\boldsymbol{n}=(\boldsymbol{k}\times\boldsymbol{k}')/(|\boldsymbol{k}\times\boldsymbol{k}'|)$, with $\boldsymbol{k}$, $\boldsymbol{k}'$ being pion initial and final momenta. We consider 8 resonances, S_{11}, S_{31}, P_{11}, P_{13}, P_{31}, P_{33}, D_{13}, D_{15} for this amplitude. The resonance parameters are taken from the phase shift analyses of $\pi-N$ scattering.[25] Pion interaction in oxygen is

$$f_{2T,2J}(^{16}\mathrm{O})=f_{2T,2J}\times\left\{1-\frac{2f^l_{2T,2J}}{\pi}\times C\times\int k^2\,\mathrm{d}k\,Q_0(k,K)G_0(k,E)\right\}^{-1}.$$

$$C=1+(k^2+m_\pi^2)^{1/2}/(k^2+m_N^2)^{1/2},$$

$$Q_0(k,K)=\begin{cases}1 & \text{if}\quad \xi K+k<P_F,\\ 0 & \text{if}\quad |\xi K-k|>P_F,\\ [P_F^2-(\xi K-k)]/4\xi Kk & \text{otherwise,}\end{cases}$$

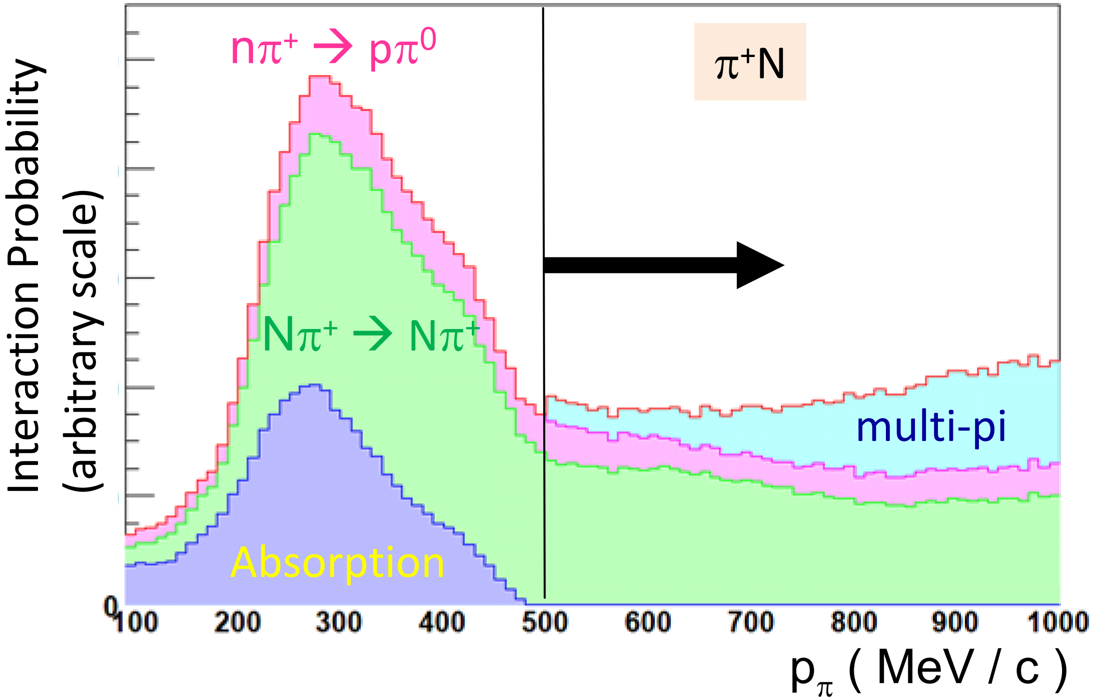
$$G_0^{-1}(k,E)=\left(E-\frac{k^2}{2M}\right)^2-k^2-m_\pi^2+i\varepsilon,$$

$$\xi=1\Big/\left(1+\frac{W}{m_N}\right),$$

11. Nuclear effects (Final state interactions of hadrons)

Higher momentum ($p_\pi \geq 500$MeV/c),

π - N interaction cross-section is used.



Mean multiplicity

$\pi^+ p$ $1.94 + 0.28 * \log(s) + 0.130 * (\log(s))^2$

$\pi^- p$ $0.59 + 0.81 * \log(s) + 0.074 * (\log(s))^2$

s : Total energy

Actual multiplicity is calculated from

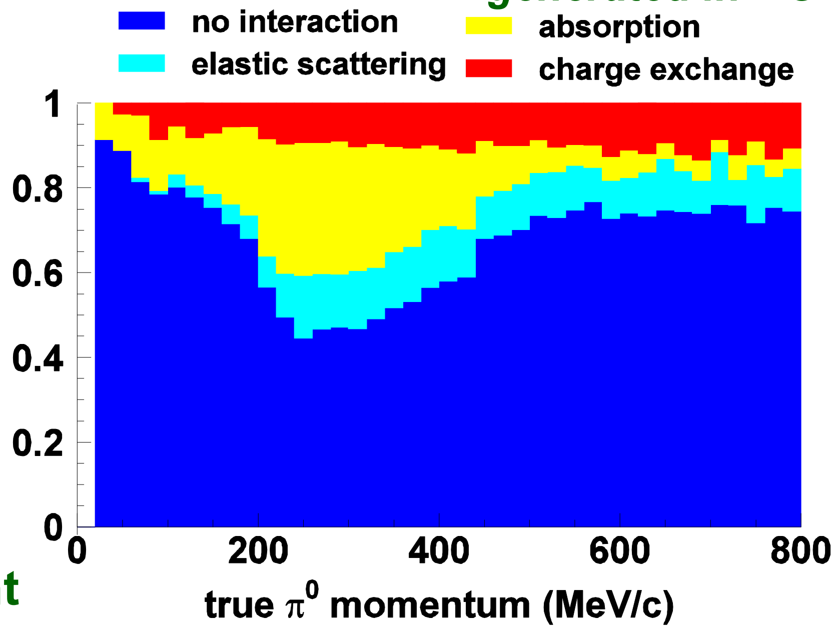
J.Whitmore Phys. Rep. 27C (1976) 187.

11. Nuclear effects (Final state interactions of hadrons)

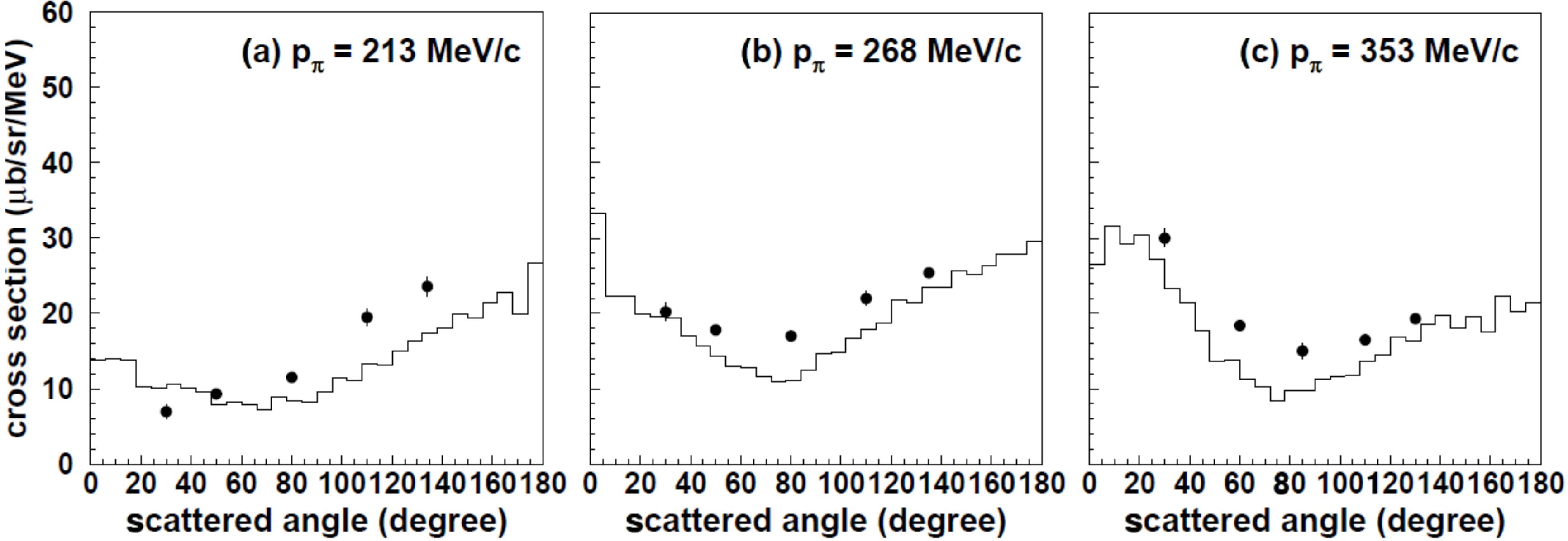
Checked with π^+ ^{16}O scattering
or photo - π production
experiments.

Monte-Carlo simulation reproduces
various distributions quite well.

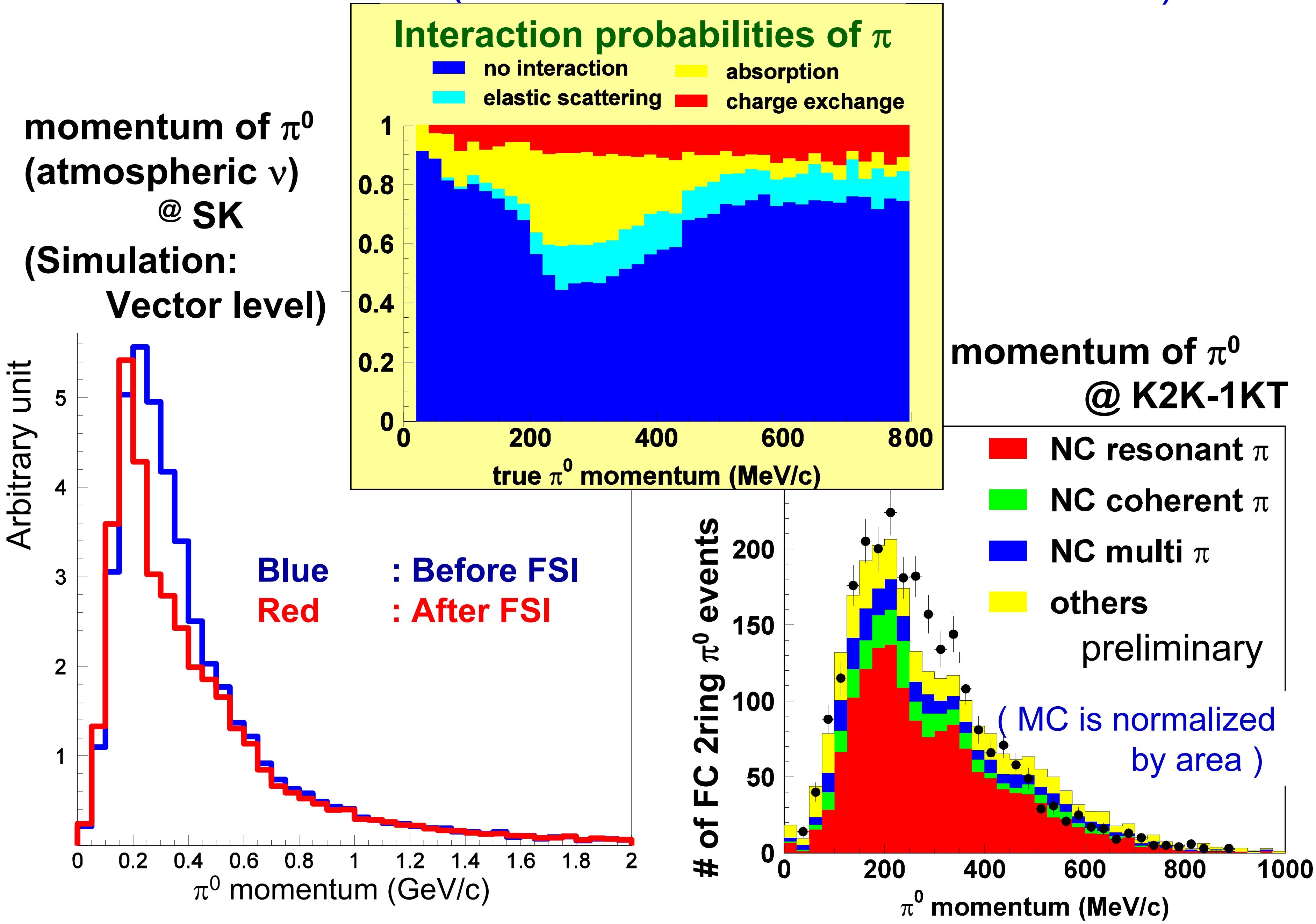
Interaction probabilities of π
generated in ^{16}O



Comparison with π^+ ^{16}O scattering experiment



11. Nuclear effects (Final state interactions of hadrons)



momentum of π^0 (atmospheric ν) @ SK (Simulation: Vector level)

Interaction probabilities of π

momentum of π^0 @ K2K-1KT

Blue : Before FSI

Red : After FSI

preliminary

(MC is normalized by area)

11. Nuclear effects (Final state interactions of hadrons)

Interactions of η

$\eta + N \rightarrow N^{*} \rightarrow N + X \qquad X : \eta, \pi, \pi\pi$

Interaction cross-section : Use Breit - Wigner formula

$$\sigma(k) = \frac{\pi}{k^2} \cdot \frac{\Gamma_{\eta N}\Gamma_X}{(W - M_N)^2 + \Gamma_{tot}^2 / 4}$$

Γ_{tot} : Total width of N*

$\Gamma_{\eta N}$: Partial decay width of N* → η + N

$\Gamma_{\eta X}$: Partial decay width of N* → X

As the intermediate resonance (N^*),
N(1535) and N(1650) are considered.

The direction of the scattered η is isotropic in the resonance rest frame.

ω is also simulated in the same manner as η.

11. Nuclear effects (Final state interactions of hadrons)

Nucleon re-scattering

Originally prepared by the members of IMB group. (W. Gajewski for K2K)

Nucleon re-scattering is also simulated by using the cascade model.

Elastic scattering, single π and two π productions are considered.

(Original ref. S.J.Lindenbaum and R.M.Sternheimer, Phys.Rev. 105 (1957),
Modifications in MECC7 and GCALOR have been taken into account.)

$N + N \rightarrow N + N$

$N + N \rightarrow N^* + N$

$N + N \rightarrow N^* + N^*$

$N^* \rightarrow N' + \pi$

Interaction probabilities (tables of each interaction mean free path) and direction of nucleons or intermediate resonances are taken from MECC7 and GCALOR.

(Basic parameters are taken from various experiments.)

Decay of the intermediate resonances
Isotropic in the resonance rest frame.

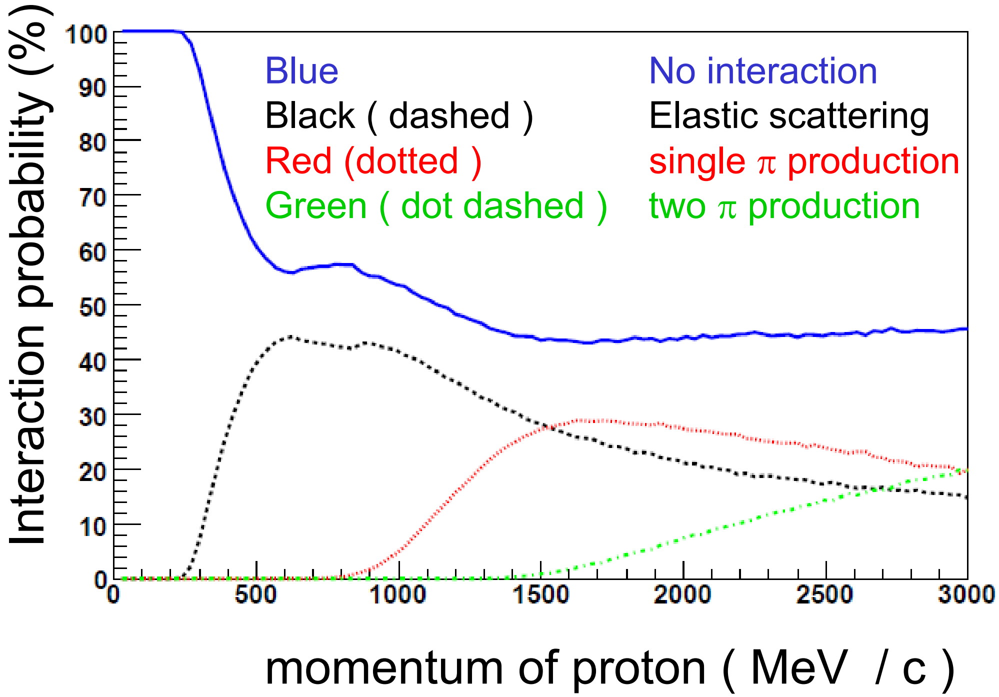
11. Nuclear effects (Final state interactions of hadrons)

Nucleon re-scattering

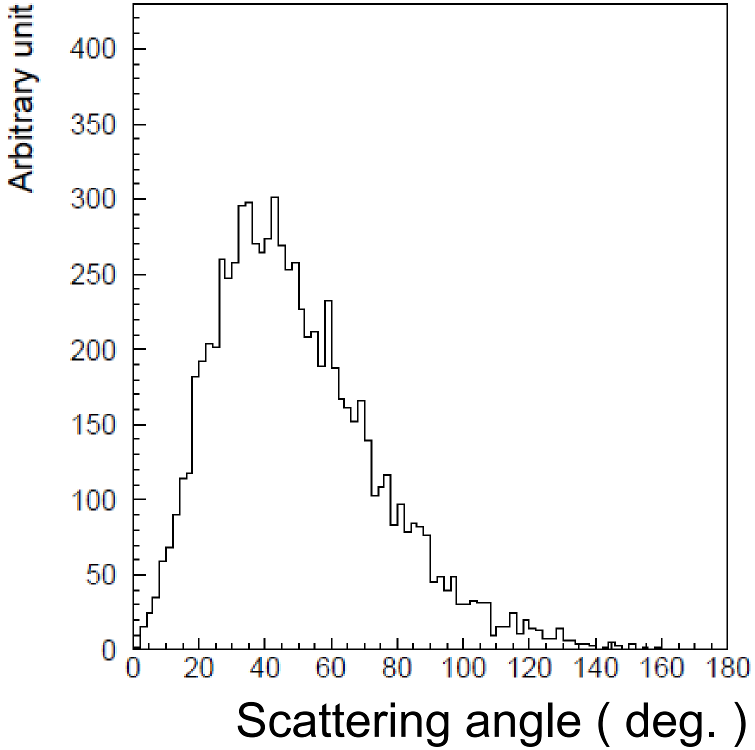
Some distributions (results of the simulation)

Target : ^{16}O

Momentum dependence
of the interaction probabilities of proton



Scattering angle of the outgoing nucleon
(most energetic one)
Incident momentum of proton
0.5 ~ 1.0 GeV / c (uniform)



12. Remaining Issues

Nucleon emission after pion absorption

From the experience of SciBooNE,
we realized that the nucleon emission
after the pion absorption seems to be visible
with the full active detectors.

Need to incorporate various recent precise models.

From dipole form factors to more precise ones.

Use spectrum functions instead of the simplest Fermi gas mode.

More proper treatment of resonances in meduim.
Nuclear medium correction
in the meson production via resonance.

Another resonance production models.

Improved treatments of the higher momentum pion in nuclear medium.

Fin.

Sample program of NEUT

```
neut_ntpl [ CARD FILE ]  [ output NTUPLE file ]
```

The card file controls the energy distribution, direction, target etc..

In the following pages, there are some information how to control the program.

NEUT card file

```
C--------------------------------
C Number of events ; EVCT-NEVT
EVCT-NEVT  1000
C--------------------------------
C Particle Code ; EVCT-IDPT
EVCT-IDPT 14
C--------------------------------
C fixed  VERTEX ; EVCT-MPOS 1
C random VERTEX ; EVCT-MPOS 2
C
C EVCT-MPOS 1
C VECT-POS  100. 0. 0.
C
EVCT-MPOS 2
EVCT-RAD  100.
C--------------------------------
C fixed  DIRECTION ; EVCT-MDIR 1
C random DIRECTION ; EVCT-MDIR 2
C
EVCT-MDIR 1
EVCT-DIR 0. 0. 1.
C--------------------------------
C fixed  MOMEMTUM ; EVCT-MPV 1
C random MOMEMTUM ; EVCT-MPV 2
C
C EVCT-MPV 1
C EVCT-PV  1000.
EVCT-MPV 2
EVCT-PV  0. 10000.
```

```
C **** TARGET INFORMATION ****
C
C NUMBNDN : total number of neutron
C          (e.g. CH => 6, H2O => 8, Ar => 22, Fe => 30)
C
NEUT-NUMBNDN 8
C
C NUMBNDP : total number of bound proton
C          (e.g. CH => 6, H2O => 8, Ar => 18, Fe => 26)
C
NEUT-NUMBNDP 8
C
C NUMFREP : total number of free proton
C          (e.g. CH => 1, H2O => 2, Ar => 0, Fe => 0)
C
NEUT-NUMFREP 2
C
C NUMATOM : atomic number of atom heavier than hydrogen
C          (e.g. CH => 12, H2O =>16, Ar => 40, Fe => 56)
C
NEUT-NUMATOM 16
C
NEUT-PFSURF 0.225
NEUT-PFMAX  0.225
C
```

NEUT card file

```
C **** NEUTRINO INTERACTION ****
C
C FERM : Fermi motion  0 : on ( default )  1 : off
C
NEUT-FERM 0
C
C PAUL : Pauli blocking  0 : on ( default )  1 : off
C
NEUT-PAUL 0
C
C NEFF : Nuclear effect in O16  0 : on ( default )  1 : off
C
NEUT-NEFF 0
C
C IFORMLEN : Formation zone 1: on (default) 0: off
C
NEUT-IFORMLEN 1
C
C Nucleon rescattering
C
C NUCRES-RESCAT  1: on (default) 0: off
C
NUCRES-RESCAT 1
C
C NUCRES-XNUCFACT
C   cross-section factor to study uncertainty default = 1.
C
NUCRES-FACT 1.
```

```
C
C  PDF for DIS is set in this section
C  (GRV94DI -> 7, GRV98_LO -> 12)
NEUT-PDF    12
C  PDF Correction is used?
C              ( original=0, modified=1)
NEUT-BODEK   1
C  Select Coherent pion model
C              Rein & Sehgal             =0(default)
C              Kartavtsev et al.         =1
NEUT-COHEPI  0
C
```

```
C RAND : random seed
C      0 : Read RANDOM number from FILE
C      1 : Generating RANDOM SEED from the time
C
NEUT-RAND 1
```

NEUT card file

```
C MODE : Interaction mode
C           0 : normal ( default )
C          -1 : input cross section by NEUT-CRS / NEUT-CRSB
C           n : sellect one mode ( n > 0 )
C
NEUT-MODE 0
C
C CRS  :  Multiplied factor to cross section on each mode. ( neu )
C CSRB :  Multiplied factor to cross section on each mode. ( neu-bar )
C
C                 1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27
NEUT-CRS    1. 1. 1. 1. 1. 1. 1. 1. 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1. 1.
NEUT-CRSB  1. 1. 1. 1. 1. 1. 1. 1. 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1. 1.
C              nu                             nub
C  1           CC Q.E.
C  2-4:        CC 1pi
C  5:          CC DIS 1320
C  6-9:         NC 1pi
C  10:         NC DIS 1320
C  11-13       NC els                         11-14:NC els
C  14,15:      coherent                       15,16:
C  16:         CC eta                         17:
C  17,18:      NC eta                         18,19
C  19:         CC K                           20:
C  20,21:      NC K                           21,22:
C  22:         dummy
C  23:         CC DIS
C  24:         NC DIS
```

Interaction modes used in NEUT

######### NEUTRINO MODE #########
***** CHARGED CURRENT *****

-- ELASTIC --
1 : NEU,N --> LEPTON-,P

-- SINGLE PI FROM DELTA RESONANCE --
11 : NEU,P --> LEPTON-,P,PI+
12 : NEU,N --> LEPTON-,P,PI0
13 : NEU,N --> LEPTON-,N,PI+

16 : NEU,O(16) --> LEPTON-,O(16),PI+

-- SINGLE GAMMA FROM DELTA RESONANCE --
17 : NEU,N --> LEPTON-,P,GAMMA

-- MULTI PI (1.3 < W < 2.0 GeV) --
21 : NEU,(N OR P) --> LEPTON-,(N OR P),MULTI PI

-- SINGLE ETA FROM DELTA RESONANCE --
(added 97/12/01 J.Kameda)
22 : NEU,N --> LEPTON-,P,ETA0

-- SINGLE K FROM DELTA RESONANCE --
(added 98/02/25 J.Kameda)
23 : NEU,N --> LEPTON-,LAMBDA,K+

-- DEEP INELASTIC (2.0 GeV < W , JET set) --
26 : NEU,(N OR P) --> LEPTON-,(N OR P),MESONS

***** NEUTAL CURRENT *****

-- SINGLE PI FROM DELTA RESONANCE --
31 : NEU,N --> NEU,N,PI0
32 : NEU,P --> NEU,P,PI0
33 : NEU,N --> NEU,P,PI-
34 : NEU,P --> NEU,N,PI+

36 : NEU,O(16) --> NEU,O(16),PI0

-- SINGLE GAMMA FROM DELTA RESONANCE --
38 : NEU,N --> NEU,N,GAMMA
39 : NEU,P --> NEU,P,GAMMA

-- MULTI PI (1.3 GeV < W < 2.0 GeV) --
41 : NEU,(N OR P) --> NEU,(N OR P),MULTI PI

-- SINGLE ETA FROM DELTA RESONANCE --
(added 97/12/01 J.Kameda)
42 : NEU,N --> NEU,N,ETA0
43 : NEU,P --> NEU,P,ETA0

52 : NEU,N --> NEU,N
-- SINGLE K FROM DELTA RESONANCE --
(added 98/02/20 J.Kameda)
44 : NEU,N --> NEU,LAMBDA,K0
45 : NEU,P --> NEU,LAMBDA,K+

-- DEEP INELASTIC (2.0 GeV < W , JET set) --
46 : NEU,(N OR P) --> NEU,(N OR P),MESONS

-- ELASTIC --
51 : NEU,P --> NEU,P

Interaction modes used in NEUT

```
######### ANTI NEUTRINO MODE #########

***** CHARGED CURRENT *****

      -- ELASTIC --
  -1 : NEUBAR,P --> LEPTON+,N

      -- SINGLE PI FROM DELTA RESONANCE --
 -11 : NEUBAR,N --> LEPTON+,N,PI-
 -12 : NEUBAR,P --> LEPTON+,N,PI0
 -13 : NEUBAR,P --> LEPTON+,P,PI-

 -16 : NEUBAR,O(16) --> LEPTON+,O(16),PI-

      -- SINGLE GAMMA FROM DELTA RESONANCE --
 -17 : NEUBAR,P --> LEPTON+,N,GAMMA

      -- MULTI PI (W > 1.4 GEV) --
 -21 : NEUBAR,(N OR P) --> LEPTON+,(N OR P),MULTI PI

      -- SINGLE ETA FROM DELTA RESONANCE --
                     (added 97/12/01 J.Kameda)
 -22 : NEUBAR,P --> LEPTON+,N,ETA0

      -- SINGLE  K FROM DELTA RESONANCE --
                     (added 98/02/25 J.Kameda)
 -23 : NEUBAR,P --> LEPTON+,LAMBDA,K0

      -- DEEP INELASTIC (2.0 GeV < W , JET set) --
 -26 : NEUBAR,(N OR P) --> LEPTON+,(N OR P),MESONS
```

```
    ** NEUTAL CURRENT **

    -- SINGLE PI FROM DELTA RESONANCE --
-31 : NEUBAR,N --> NEUBAR,N,PI0
-32 : NEUBAR,P --> NEUBAR,P,PI0
-33 : NEUBAR,N --> NEUBAR,P,PI-
-34 : NEUBAR,P --> NEUBAR,N,PI+

-36 : NEUBAR,O(16) --> NEUBAR,O(16),PI0

    -- SINGLE GAMMA FROM DELTA RESONANCE --
-38 : NEUBAR,N --> NEUBAR,N,GAMMA
-39 : NEUBAR,P --> NEUBAR,P,GAMMA

    -- MULTI PI (W > 1.4 GEV) --
-41 : NEUBAR,(N OR P) --> NEUBAR,(N OR P),MULTI PI

    -- SINGLE ETA FROM DELTA RESONANCE --
                   (added 97/12/01 J.Kameda)
-42 : NEUBAR,N --> NEUBAR,N,ETA0
-43 : NEUBAR,P --> NEUBAR,P,ETA0

    -- SINGLE  K  FROM DELTA RESONANCE --
                   (added 98/02/20 J.Kameda)
 -44 : NEUBAR,N --> NEUBAR,LAMBDA,K0
 -45 : NEUBAR,P --> NEUBAR,LAMBDA,K+

    -- DEEP INELASTIC (2.0 GeV < W , JET set) --
-46 : NEUBAR,(N OR P) --> NEUBAR,(N OR P),MESONS

    -- ELASTIC --
-51 : NEUBAR,P --> NEUBAR,P
-52 : NEUBAR,N --> NEUBAR,N
```

Meanings of the variables in ntuple

MODE : Interaction mode
NUMNU : Number of particles (including initial neutrino and nucleon)
IPNU : Particle Code
ABSPNU : Absolute momentum of particle (GeV / c)
PNU : Momentum of particle (GeV / c)

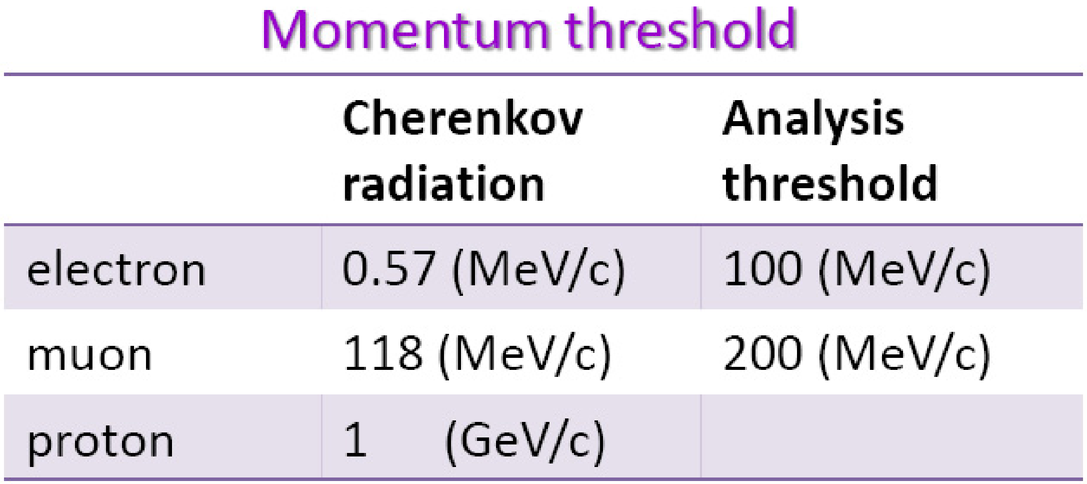
NPAR : Number of particles (after the final state interactions)
IPV : Particle code
IORGV : Parent particle
ICRNV : Tracking flag (set to 0 if the particle was interacted, absorbed or disappeared)
IFLGV : Interaction code (If the particle already interacted with the oters, this flag is set.)
- 0 : Not interacted yet.
- 1 : Decayed to the other particles
- 2 : N/A
- 3 : Absorbed
- 4 : Charge exchanged
- 5 : Stopped
- 6 : N/A
- 7 : Particle production
- 8 : Scattered

ABSPV : Absolute momentum of particle
PMOMV : Momentum of particle

Water Chernkov detector

High efficiency in detecting electrons
low momentum neutral pi0

Relatively low threshold in detecting muon and charged pion



Momentum threshold

	Cherenkov radiation	Analysis threshold
electron	0.57 (MeV/c)	100 (MeV/c)
muon	118 (MeV/c)	200 (MeV/c)
proton	1 (GeV/c)	

Precise particle type identification
showering (e-like) electron, gamma
non-showering (mu-like) muon, charged pions, etc.

PID performance

Detection threshold is high for the massive particles like proton

Charge Identification is not possible

Konstantin S. Kuzmin et al. Eur.Phys.J.C54:517-538,2008

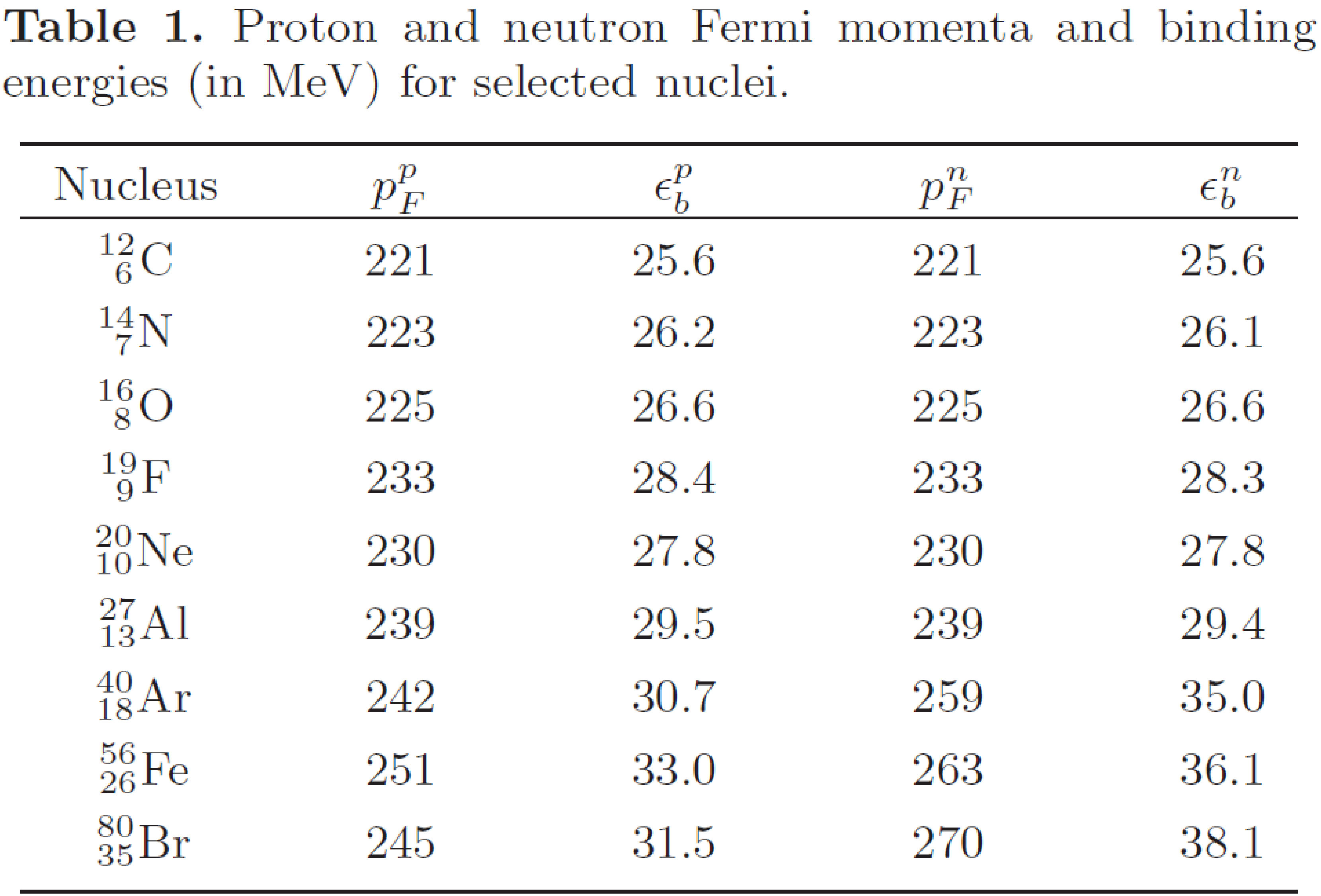
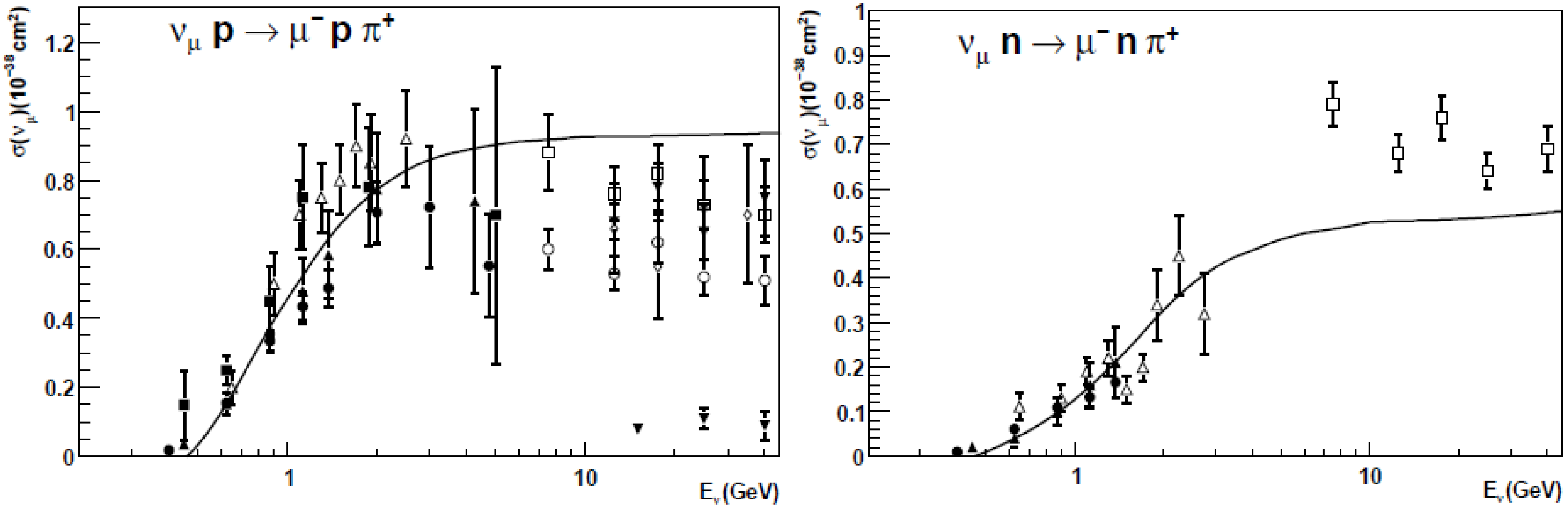


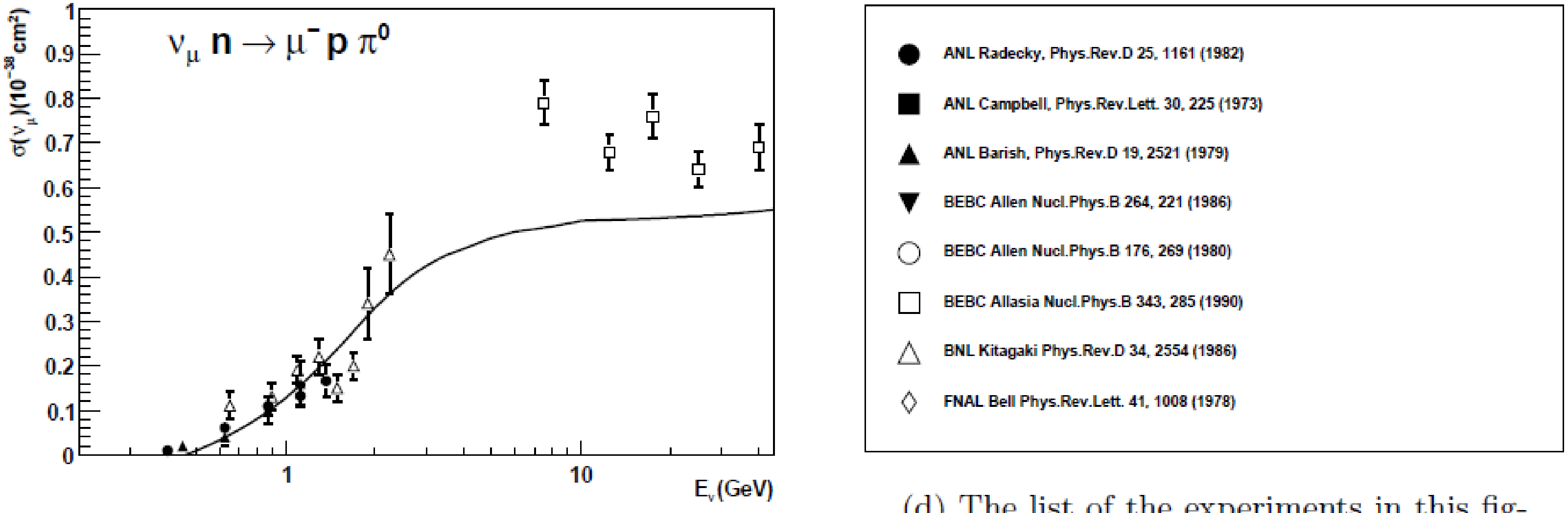
Table 1. Proton and neutron Fermi momenta and binding energies (in MeV) for selected nuclei.

Nucleus	p_F^p	ϵ_b^p	p_F^n	ϵ_b^n
${}^{12}_{6}\mathrm{C}$	221	25.6	221	25.6
${}^{14}_{7}\mathrm{N}$	223	26.2	223	26.1
${}^{16}_{8}\mathrm{O}$	225	26.6	225	26.6
${}^{19}_{9}\mathrm{F}$	233	28.4	233	28.3
${}^{20}_{10}\mathrm{Ne}$	230	27.8	230	27.8
${}^{27}_{13}\mathrm{Al}$	239	29.5	239	29.4
${}^{40}_{18}\mathrm{Ar}$	242	30.7	259	35.0
${}^{56}_{26}\mathrm{Fe}$	251	33.0	263	36.1
${}^{80}_{35}\mathrm{Br}$	245	31.5	270	38.1



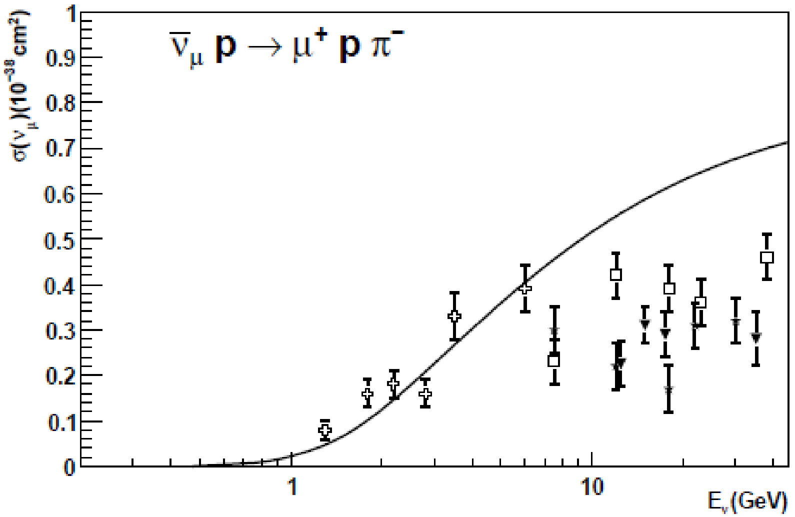
(a) $\nu_\mu p \longrightarrow \mu^- p\pi^+$

(b) $\nu_\mu n \longrightarrow \mu^- n\pi^+$

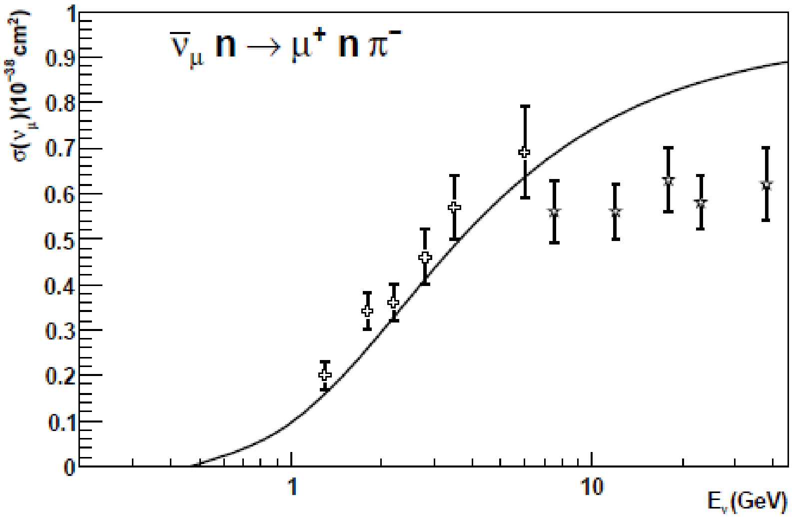


(c) $\nu_\mu n \longrightarrow \mu^- p\pi^0$

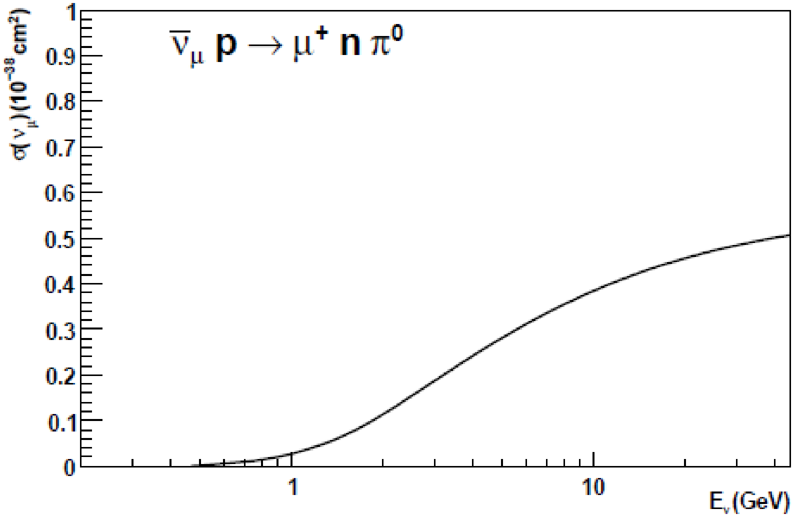
(d) The list of the experiments in this figure



(a) $\overline{\nu}_\mu p \to \mu^+ p \pi^-$



(b) $\overline{\nu}_\mu n \to \mu^+ n \pi^-$



(c) $\overline{\nu}_\mu p \to \mu^+ n \pi^0$

▼ BEBC Allen Nucl.Phys.B 264, 221 (1986)

□ BEBC Allasia Nucl.Phys.B 343, 285 (1990)

✚ P. Schreiner, GGM, Proceedings of 1979 symposium at Fermilab

★ FNAL Barish Phys.Lett.B 91, 161 (1980)

☆ BEBC Allisia, Z.Phys.C 20, 95 (1983)

(d) The list of the experiments in this figure

6. Deep Inelastic scattering

ν + N → l + hadrons

Corrections proposed by Bodek and Yang

(hep-ex/0203009, hep-ex/0308007)

1. Bjorken scaling x→ x_w

$$x_w = x \frac{Q^2 + B}{Q^2 + Ax}$$

A : target mass effect
higher twist effect
B : photoproduction limit(Q^2=0)

2. Correction to the structure function F_2

$$F_2(x) \to \frac{Q^2}{Q^2 + C} F_2(x_w)$$

to fit both intermediate-x and low-x

3. d/u ratio

d_v → $d'_v(d_v,u_v)$
u_v → $u'_v(d_v,u_v)$

Correction to the conversion from F_2^d to F_2^n

4. Longitudinal R

$$2xF_1 = F_2 \frac{1 + 4Mx^2/Q^2}{1 + R}$$

Corrections for the spin of the target.

These correction parameters are obtained by fitting various existing experimental results.

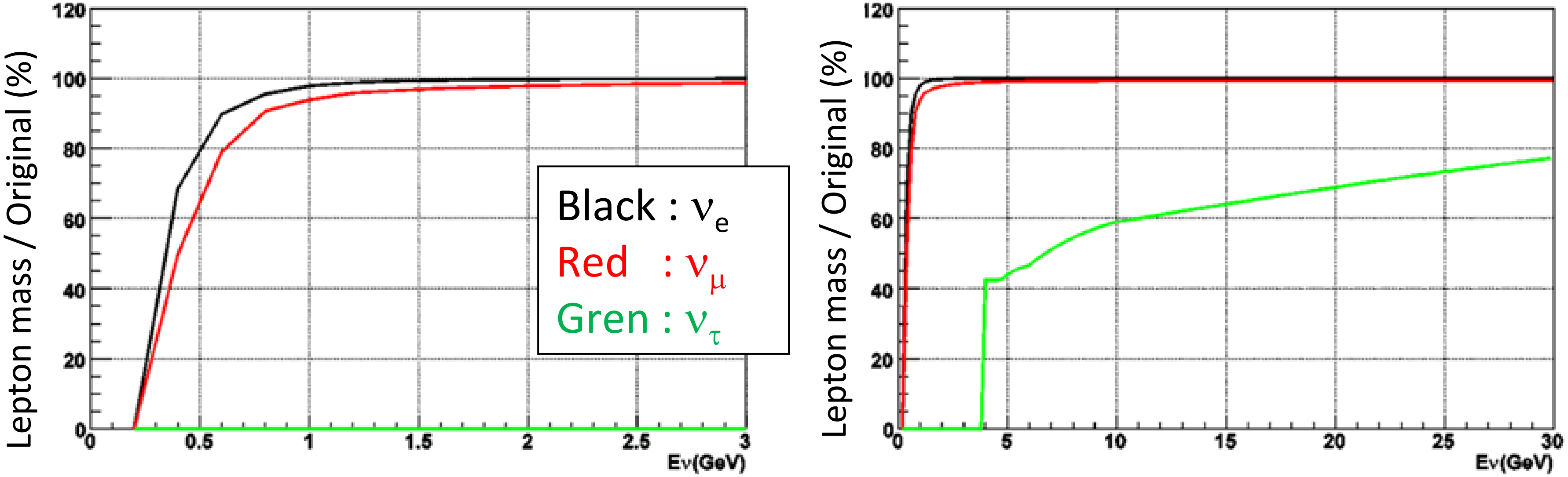
4. Single meson production via resonances

(G.Mitsuka)

Simulation result -Integrated cross section-

Berger and Sehgal add the pion-pole term in the hadronic current.
This effect is the almost same as in the coherent-π production
hep-ph/0709.4378

$\nu_\mu p \rightarrow \mu^- \pi^+ p$

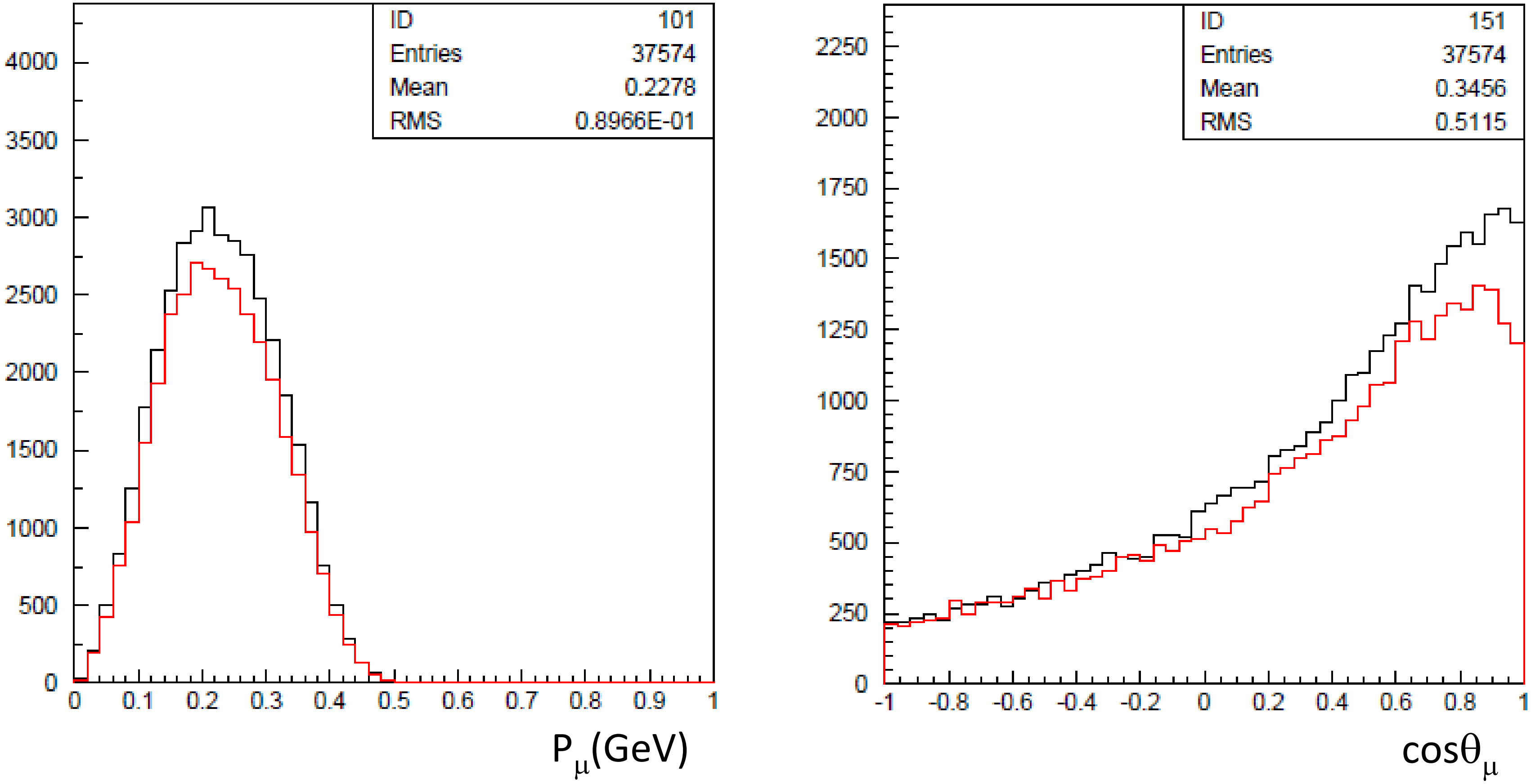


An amount of reduction depends on lepton flavor and energy
These values are consistent with the description in references

4. Single meson production via resonances

(G.Mitsuka)

Simulation result -Kinematics-



Lepton momentum has no significant change

Lepton scattering angle has suppression in forward direction

Pion and nucleon kinematics are not changed

$$\frac{\sigma(\nu N \to \nu X)}{\sigma(\nu N \to \mu^- X)} = \begin{cases} 0.26 & (\ E_\nu < 3\,\mathrm{GeV}\ ) \\ 0.26 + 0.04\,(E_\nu/3 - 1) & (\ 3\,\mathrm{GeV} \leq E_\nu < 6\,\mathrm{GeV}\ ) \\ 0.30 & (\ E_\nu \geq 6\,\mathrm{GeV}\ ) \end{cases} \tag{4.21}$$

$$\frac{\sigma(\bar{\nu} N \to \bar{\nu} X)}{\sigma(\bar{\nu} N \to \mu^+ X)} = \begin{cases} 0.39 & (\ E_\nu < 3\,\mathrm{GeV}\ ) \\ 0.39 - 0.02\,(E_\nu/3 - 1) & (\ 3\,\mathrm{GeV} \leq E_\nu < 6\,\mathrm{GeV}\ ) \\ 0.37 & (\ E_\nu \geq 6\,\mathrm{GeV}\ ) \end{cases} \tag{4.22}$$

The kinematics of the hadronic system is simulated by two different methods according to the range of invariant mass. In the region of $1.3\,\mathrm{GeV}/c^2 < W < 2.0\,\mathrm{GeV}/c^2$, only pions are considered as outgoing mesons. The mean multiplicity of pions is estimated from the result of Fermilab 15-foot hydrogen bubble chamber experiment [129] :

$$\langle n_\pi \rangle = 0.09 + 1.83\ln(W^2) \tag{4.23}$$

The number of pions in each event is determined by using KNO (Koba-Nielsen-Olsen) scaling. Since the range of W overlaps with that in single pion production, $n_\pi \geq 2$ is required in this W region. The forward-backward asymmetry of pion multiplicity in the hadronic center of mass system is included using the results from BEBC experiment [130] :

$$\frac{n_\pi^F}{n_\pi^B} = \frac{0.35 + 0.41\ln(W^2)}{0.5 + 0.09\ln(W^2)} \tag{4.24}$$